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An Alternative Endogenous Inefficiency Structure in Stochastic Frontier Analysis under Soft Budget

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Abstract

This note presents a hospital cost inefficiency index model as a consequence of some subjective equilibrium. We model the hospital subsidy depends on its scale, not the deficit. With a simplified model specification in which the role of subsidy is emphasized, the subsidy per deficit forms the distribution of cost inefficiency which corresponds to the inefficiency term in stochastic frontier analysis. We check the distribution with actual data controlling the location of the hospitals (a key source of inefficiency in many previous analyses).

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1 Introduction

This note is an attempt to reconsider the distributional assumption of inefficiency in stochastic frontier analysis (SFA). In SFA, half-normal distribution is often used to capture inefficiency, while some works consider positive mode models (for example, [Greene, 1990]; [Fujii, 2001] for hospital inefficiency analysis). The distributional assumption on the inefficiency is argued because it summarizes the mechanism through which the inefficiency is generated and observed.

To do a preliminary analysis to build an appropriate inefficiency distribution assumption in municipal hospitals case, [Fujii, 2016] introduced [Sano, 2007]'s hospital behavior model together with the soft-budget problem ([Kornai, 1986]) discussed in [Yamauchi, 2009]. The model sets the government subsidy to a hospital as a function of its deficit and found positive mode distributions are more suitable as an inefficiency distribution.

On the other hand, the approach seems somewhat like short-run settings. There is also a long-run problem: The geometric distribution of public hospitals in Japan. Sometimes municipal hospitals are regarded as operating on larger scales than their optimum ([Fuj and Ohta, 1999], [Otani and Fukuda, 2019]). In such a situation, the mechanism through which subsidy directly stimulates scale should be considered. The subsidy is necessary to maintain the healthcare provision system, while it becomes a major financial concern in Japan ([Kawaguchi et al., 2014]). So quantitative method to understand the relation in SFA is required, even if the effect of the subsidy is not just "inefficiency."

Also, there have been advances in the non-parametric estimation of SFA (say, [Kneip et al., 2015], [Forchini and Theler, 2023]). These analytical techniques enable us to relax our assumption on the inefficiency distribution and to encompass specific structures in a specific industry (like soft-budget in public entities).

To prepare such an analysis, we introduce in this note an alternative model of the soft-budget problem that emphasizes the relationship between scale and subsidy. The model is very simple, so does not require statistical estimation.

2 Methodology

As an example of a municipal hospital behavior model, let us consider a modified version of [Fujii, 2016]. [Fujii, 2016] utilizes a simplified model of [Sano, 2007], which emphasizes the effect of subsidy. Our model is written in a general form:

Maximize

$$U(B, \Pi^S) \tag{1}$$

with respect to B subject to

$$\Pi^S = \Pi(B) + S(B). \tag{2}$$

The hospital is assumed to maximize its utility function (1) defined over the space of scale variable (number of beds, B) and the net income Π^S (gross profit $\Pi(B)$ plus expected subsidy $S(B)$). To encompass the idea of a soft-budget problem ([Kornai, 1986], [Yamauchi, 2009]), each hospital is assumed to know the level of S in advance. B stands for the quantitative aspects of healthcare provision by the municipal hospital, whereas Π^S appears as an argument of utility because each hospital is a financially independent entity.

The difference between our model and [Fujii, 2016] is that we assume S depends on B whereas S depends only on the deficit in [Fujii, 2016]. Our model relatively emphasizes the role of public hospitals in a regional area, while [Fujii, 2016] focuses mainly on the financial soft-budget problem.

The optimality condition of the hospital's subjective equilibrium is

$$\frac{\partial U}{\partial B} + \frac{\partial U}{\partial \Pi^S} \left\{ \frac{\mathrm{d}\Pi}{\mathrm{d}B} + \frac{\mathrm{d}S}{\mathrm{d}B} \right\} = 0. \tag{3}$$

This equation implies that the production scale is expanded if a subsidy is enhanced.

In order to understand quantitatively, [Sano, 2007] assumes a specific functional form. We adopt the following specification:

$$\Pi = pB - rB,\ S(B) = sB,\ U = \alpha \log B + \Pi^S, \tag{4}$$

where pB is the expected revenue; rB is the cost that depends on B; α, s, p and r are non-negative constants.

We assume the observed values of B and the cost C are determined in the equilibrium and is characterized as:

$$B^*(s) = \frac{\alpha}{r-p-s},\ C^*(s) = rB^* = \frac{\alpha r}{r-p-s}. \tag{5}$$

The inefficiency index $I \in (0,1)$ is defined as:

$$I = \frac{C^*(s) - C^*(0)}{C^*(s)} = \frac{s}{r-p}. \tag{6}$$

If the cost is observed as $C^*(s) = C^*(0)e^u$, where u is the non-negative inefficiency term as modeled in SFA, then the value of u can be computed as:

$$u = -\log(1-I). \tag{7}$$

We check the distribution of u using data from *Annual Statistics of Public Enterprises (Chihou Kouei-Kigyou Nenkan in Japanese)* Vol 65, edited by Local Public Financial Bureau, Ministry of Internal Affairs and Communications, the Government of Japan. This source reports data for the 2017 fiscal year. We limit our sample to operating hospitals that receive subsidies, that are located in an unprofitable area, and that have a net deficit, resulting in a sample of 738 hospitals. We also excluded those hospitals with $I \geq 1$, since our functional specification cannot approximate their behavior. The final sample size is 570. The descriptive statistics are shown in Table 2.

Location	Mean	Std. Dev.	Min.	Max.
Number of Beds B	234.714	176.620	25	880
Subsidy per bed S/B (in thousand yen)	2884.44	2350.166	32.49495	20448.8
Inefficiency term u	1.286996	.914121	.0286657	7.027087
Inefficiency index I	.6367521	.2135114	.0282587	.9991125

Table 1: Descriptive Statistics

3 Result and Comments

Table 2 summarizes the data used in our analysis. The mean of I is as high as 0.64, which may be a result of limiting our sample to the unprofitable areas. Although the soft-budget mechanism is the only source of inefficiency considered in our model, the result implies the significance of the problem. For empirical applications, the shape of the distribution is extremely helpful. Figure 3 illustrates the distribution of u computed by (7). Clearly, we observe a positive mode for the distribution. This result suggests we better consider positive mode inefficiency for SFA of municipal hospitals in Japan. Parametric as well as non-parametric approaches, such as [Forchini and Theler, 2023] or [Kneip et al., 2015] seem applicable in our case.

There are some drawbacks to the approach shown in this note, however. We observed about 23% loss in sample size ($1 - 570/738$) by imposing $I < 1$ restriction on the sample. Looking at (6), the linearity assumption of the subsidy-bed relation (6) may have caused the loss. Also, the absence of fixed cost and scale economy structure should be treated for empirical application. Nonetheless, the simplicity of our model well depicts the distributional assumption of inefficiency under soft-budget circumstances.

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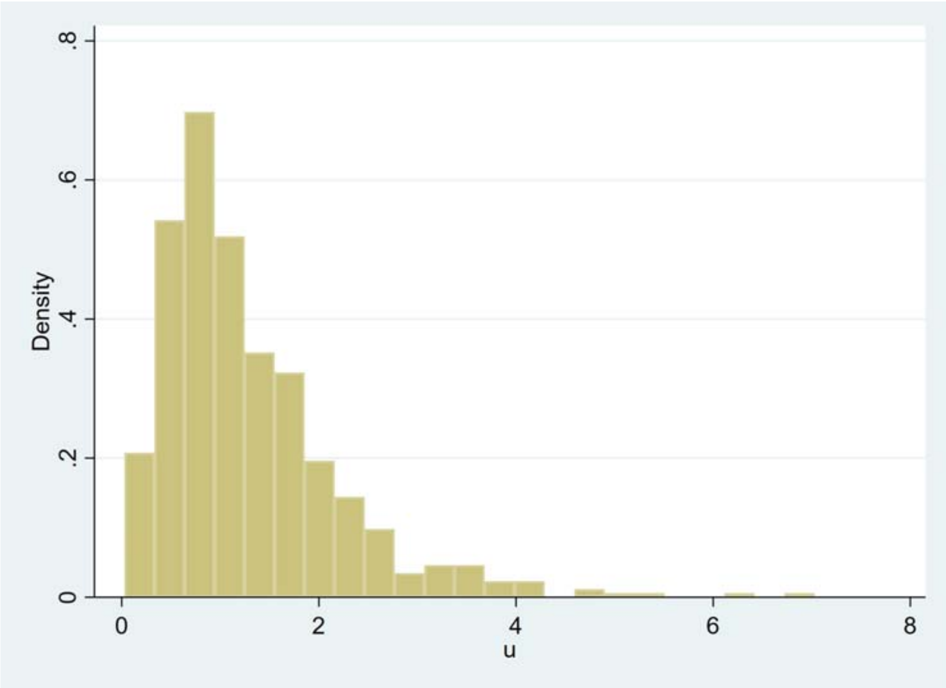

Figure 1: Distribution of u

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