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# On Keynes' Business Cycle Theory

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**Abstract**

The purpose of this paper is to formulate Keynes' own view regarding the business cycle as simply and faithfully as possible by using a simple macro-dynamic model based on the intertemporal optimization of the firm's investment decision under demand constraint. We demonstrate that under a specific assumption regarding the firm's expectation modification behavior Keynes' view that economic fluctuations are caused by optimistic soars in the marginal efficiency of capital and their inevitable collapses can be faithfully formulated.

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## 1. Introduction

The objective of this paper is to formulate Keynes' own view regarding the business cycle as simply and faithfully as possible. In Chapter 22 of *General Theory* (Keynes (1936)) he summarizes his view of business cycle as follows:

(1) The most important factor that generates the business cycles is fluctuation in "the marginal efficiency of capital (MEC, hereafter)". The move of investment caused by the fluctuation of MEC brings about the business cycles through the principle of effective demand.

(2) A boom is mainly caused by excessive investment driven by an optimistic soar in the MEC. Here, "excessive investment" is interpreted according to Keynes' own words as a type of investment that is "made in conditions which are unstable and can not endure because it is prompted by expectations which are destined to disappointment".

(3) The main reason why the economy recovers regularly from recessions is that capital stock over-accumulated during a boom naturally depreciates and returns to its normal level within several years.[1]

This Keynes' view of the business cycle seems to be very convincing, but this view has not necessarily been formulated properly even in the context of post Keynesian business cycle theories.[2] In the post Keynesian school, there are various types of business cycle models. For example, Samuelson (1939), Hicks (1950) and Goodwin (1951) presented business cycle models based on the acceleration principle, i.e., the assumption that investment depends on the current changes in aggregate income. Kalecki (1935, 1937)

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[1] Keynes also pointed out the existence of inventory cost as a reason for regular recoveries from recessions.

[2] There seems to be little research that attempts to formulate faithfully this Keynes' view on business cycles in the mainstream new Keynesian (NK) framework. Recently, Branch and McGough (2010), Branch and Evans (2011), De Grauwe (2011, 2012) and so on introduced heterogeneous boundedly rational expectation formations into the NK model and formulated boom and bust cycles driven by 'animal spirits' (i.e., the waves of optimism and pessimism). However, it is questionable whether these can be interpreted as a model that faithfully reproduces Keynes's views described above. See also footnote 5 regarding this point.

constructed models built on the profit principle, i.e., the assumption that investment depends on the current aggregate profits (or the rate of profit). A model by Kaldor (1940) was built under the assumption that investment depends on the current total profit (or income) and its functional form is the sigmoid shape. Finally, in Goodwin (1967)'s model, business cycles are led by a certain income distribution structure between employees and capitalists[3]. However, Keynes himself emphasized the MEC (or the expected rate of return on capital) as the crucial determinant of investment, and these models are hardly a faithful formulation of that point.[4]

There are not many attempts to construct a Keynesian business cycle model in which the expected return on capital is the primary determinant of investment. Benassy (1984) and Murakami (2018, 2019, 2020) are two such attempts. Benassy (1984) constructed a short-run non-Walrasian model in which the wage is adjusted in response to the unemployment level, and the expected demand as a determinant of investment is adjusted adaptively in response to its realized value. He then showed that a limit cycle exists when the effect of destabilizing quantity dynamics exceeds the effects of stabilizing price dynamics. Murakami (2018, 2019, 2020) presented a post Keynesian cyclical growth model built on the assumption that investment depends on the expected rate of return on capital and its functional form is the sigmoid shape like the Kaldor (1940)'s model. He then showed that if investment is highly elastic to the expected rate of return, and if the expected rate of return is frequently revised in response to its realized value, then the unique limit cycle exists, and an economy starting from any state other than the long-run equilibrium will converge stably to the limit cycle.

However, these models are not adequate as a formulation of Keynes' own business cycle theory in the following two respects. First, according to Keynes, the investment is determined so as to maximize the discounted present value of profits, given future

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[3] In this sense Goodwin (1967)'s model should be interpreted as a kind of "Marxian" business cycle theory. But this model was subsequently extended to the Keynesian framework where the output level is determined by the principle of effective demand by Wolfstetter (1982), Franke and Asada (1994), and so on.

[4] This holds true for more sophisticated post Keynesian macroeconomic models such as Flaschel *et al.* (1997) and Chiarella *et al.* (2005).

expectations (long-term expectations), but in these models the (dynamic) optimization behavior of the firm is not made explicit. Secondly, Keynes thought that booms are caused by the over-investment which is brought about by the optimistic expectations that are destined to disappointment, and that the economy will enter a phase of recession after the sudden collapse of such optimistic expectations. In these models, however, such discontinuous features of economic fluctuations have not been modeled faithfully.[5]

The purpose of this paper is to formulate Keynes' business cycle theory as simply and faithfully as possible. The model presented in this paper is post Keynesian in that it is based on the principle of effective demand. However, it has a feature that is not often seen in post Keynesian models in that we explicitly introduce the intertemporal optimization of the firm's investment decision. The reason why we introduce it is that the concept of MEC defined by Keynes can be well formulated using this framework as explained by Nagatani (1981, ch6). In our model the firm derives the optimal investment path under the subjective expectation regarding the aggregate demand, so the optimal investment demand in each period depends on the expected aggregate demand. Once the current investment demand is determined, the current equilibrium output is also determined based on the principle of effective demand. However, there is no guarantee that the firm's expected aggregate demand will coincide with its realized value (i.e, the equilibrium output). In this paper we assume a simple expectation modification behavior that the firm revises the expectation in response to its realized value only in the steady state, and we show that under such an assumption Keynes' business cycle theory can be formulated both simply and faithfully.

The remainder of this paper is organized as follows. In the next section we formulate

[5] Recently, boom and bust cycles driven by 'animal spirits' (i.e., the waves of optimism and pessimism) have been constructed by introducing heterogeneous boundedly rational expectation formation in both new Keynesian (NK) and post Keynesian (PK) schools. In NK school the references listed in footnote 1 correspond to such attempts. In PK school Franke (2012, 2014), Flaschel et.al (2018) and so on corresponds to such attempts. These studies, however, are not faithful formulations of Keynes's own business cycle theory in that they are not based on the firm's optimal investment decisions given long-term expectations.

the intertemporal optimization problem of the firm's investment decision under subjective expectations regarding the aggregate demand and the interest rate. In section 3 we investigate the macro-dynamics of the economy when subjective expectations are fixed and not revised. In section 4 we consider the case where only the aggregate demand expectation is revised in the steady state, and then reproduce the Keynes' business cycle theory as a deviation from the steady state where the expectation is self-fulfilling. In section 5 we investigate the case where the expectations of both the aggregate demand and the interest rate are revised simultaneously. Finally, we conclude the paper in section 6.

## 2. Firm's investment decision

Before constructing a macro-dynamic model, in this section we discuss the firm's investment decisions[6]. Consider an economy where only one type of final product which can be used for both consumption and investment is produced. We assume that there are many homogeneous firms and normalize their number to one without loss of generality. It is also assumed that, since the size of each firm is sufficiently small, each firm does not imagine that its own investment demand is capable of affecting the aggregate demand. At time $s$ the firm faces the following problem:

$$\max_{\{I_t\}_{t=s}^{\infty},\{K_t\}_{t=s}^{\infty}} V_s = \int_s^{\infty} [F(K_t, L_t) - w_t L_t - \{I_t + c(I_t)\}] e^{-\int_s^t r_u^e du} \, dt$$
$$\text{s.t.} \quad \dot{K}_t = I_t - \delta K_t, \; Y_t^e = F(K_t, L_t) \quad (K_s\text{: } given) \tag{1}$$

where $K_t$ is the capital input, $L_t$ the labor input, $F(K_t, L_t)$ the production function, $w_t$ the real wage (measured in terms of the final product), $I_t$ the investment expenditure, $c(I_t)$ the investment adjustment cost function, $\delta$ the depreciation rate, and $\dot{K}_t$ the time derivative of $K_t$ (i.e., $\dot{K}_t \equiv dK_t/dt$). $Y_t^e$ and $r_t^e$ are the subjective expectations about the aggregate demand and the real interest rate, which will be explained in more detail below.

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[6] We mainly follow Nagatani (1981, ch6) regarding the argument in this section. See also Grossman (1972) on the original formulation of the firm's investment behavior adopted here.

We assume that the production function $F(K_t, L_t)$ satisfies the following usual neoclassical properties:

$$F_K > 0,\ F_{KK} < 0,\ F_L > 0,\ F_{LL} < 0,\ F_{KL} > 0,$$
$$\lim_{K\to 0} F_K = \infty,\ \lim_{K\to\infty} F_K = 0,\ \lim_{L\to 0} F_L = \infty,\ \lim_{L\to\infty} F_L = 0, \tag{2}$$

where $F_K \equiv \partial F/\partial K$ and $F_{KK} \equiv \partial F_K/\partial K$. It is also assumed that the investment adjustment cost function $c(I_t)$ is strictly convex:

$$c(0) = 0,\ c_I(I_t) > 0,\ c_{II}(I_t) > 0, \tag{3}$$

where $c_I(I_t) \equiv \partial c/\partial I_t$ and $c_{II}(I_t) \equiv \partial c_I/\partial I_t$.

Problem (1) is an intertemporal optimization problem according to which the firm maximizes the present discount value of the net cash flow stream, but it mainly differs from the standard setting in that the firm in our model has subjective expectations regarding present and future aggregate demands $\{Y_t^e\}_{t=s}^{\infty}$ and aims to maximize the objective function under the demand constraints: $Y_t^e = F(K_t, L_t)$ for any $t$[7]. This constraint means that the firm produces a quantity of the final product which is equal to the expected aggregate demand. The firm is also assumed to have the subjective expectation about the interest rate in each point in time and to maximize the objective function under it. Furthermore, the wage at each point in time is assumed to be exogenous fixed in our model (namely, our model is a fixed wage model).

To simplify the analysis, we assume throughout the paper that $\{Y_t^e\}_{t=s}^{\infty}$, $\{r_t^e\}_{t=s}^{\infty}$ and $\{w_t\}_{t=s}^{\infty}$ are constant over time:

$$Y_t^e = Y^e,\ r_t^e = r^e,\ w_t = w. \tag{4}$$

Under these assumptions, the firm's optimization problem can be rewritten as:

$$\max_{\{I_t\}_{t=s}^{\infty}, \{K_t\}_{t=s}^{\infty}} V_s = \int_s^{\infty} [Y^e - wG(K_t, Y^e) - \{I_t + c(I_t)\}] e^{-r^e(t-s)}\, dt$$
$$\text{s.t.}\quad \dot{K}_t = I_t - \delta K_t \quad (K_s\text{: given}) \tag{5}$$

where the function $L_t = G(K_t, Y^e)$ can be derived from the demand constraint $Y^e = F(K_t, L_t)$ and it satisfies the following properties:

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[7] Note that the firm's expectations are subjective so there is no guarantee that they are consistent with their realized values in macroeconomic equilibrium. Regarding the formulation of the firm's expectation correction behavior, see section 4 and 5.

$$G_K < 0,\ \lim_{K\to 0} G_K = -\infty,\ \lim_{K\to\infty} G_K = 0,\ G_{KK} > 0,\ G_Y > 0,\ G_{YY} > 0,\ G_{KY} < 0. \tag{6}$$

The first order conditions of the problem (5) are

$$1 + c_I(I_t) = q_t \tag{7.a}$$

$$\dot{q}_t = (r^e + \delta)q_t + wG_K(K_t, Y^e) \tag{7.b}$$

$$\dot{K}_t = I_t - \delta K_t \tag{7.c}$$

$$\lim_{t\to\infty} e^{-r^e(t-s)}\, q_t K_t = 0 \tag{7.d}$$

where $q_t$ is the shadow price of capital. By eliminating $q_t$ in (7.a) and (7.b), the dynamic system of the optimal investment plan chosen by the firm can be derived as

$$\dot{I}_t = \frac{(r^e + \delta)[1 + c_I(I_t)] + wG_K(K_t, Y^e)}{c_{II}(I_t)}, \tag{8.a}$$

$$\dot{K}_t = I_t - \delta K_t, \tag{8.b}$$

$$\lim_{t\to\infty} e^{-r^e(t-s)}\,[1 + c_I(I_t)]K_t = 0. \tag{8.c}$$

Figure 1 shows the phase diagram of this system. The figure clearly shows that this system has a unique optimal investment path and that this path satisfies saddle-point stability.

(Figure 1 around here)

How can the "marginal efficiency of capital" (hereafter MEC), defined by Keynes (1936) as the special discount rate which equates the present discount value of the net cash flow stream from an additional investment with its investment cost, be formulated within this framework? It is well known that the marginal benefit from an addtional investment is represented by the shadow price of capital $q_s$, because by integrating (7.b) we have

$$q_s = \int_s^\infty (-wG_K)e^{-(r^e+\delta)(t-s)}dt = \int_s^\infty \frac{\partial nc_t}{\partial K_t} e^{-(r^e+\delta)(t-s)}dt, \tag{9}$$

where $nc_t \equiv Y^e - wG(K_t, Y^e) - \{I_t + c(I_t)\}$ is the net cash flow at time $t$. This $q_s$ is known as Tobin's marginal $q$, which can be interpreted roughly as a firm's stock price[8].

[8] Strictly speaking, the shadow price $q_s$ is not equal to the firm's stock price in our model. As Hayashi (1982) demonstrated, they become equal if and only if (i) the firm behaves competitively, and (ii) the investment adjustment cost function is linearly

It can be easily verified that $q_s$ decreases with the capital stock and the expected interest rate, and increases with the expected aggregate demand and the real wage:

$$q_s = q(K_s, r^e, Y^e, w). \quad \left(\frac{\partial q_s}{\partial K_s} < 0, \ \frac{\partial q_s}{\partial r^e} < 0, \ \frac{\partial q_s}{\partial Y^e} > 0, \ \frac{\partial q_s}{\partial w} > 0, \ \right) \tag{10}$$

On the other hand, since the investment cost (or more precisely, the marginal cost of the first unit investment) is given by $1 + c_I(0)$, the MEC at time $s$ can be derived as the special discount rate $R_s$ which satisfies the following equation:

$$1 + c_I(0) = q(K_s, R_s, Y^e, w). \tag{11}$$

It can easily be confirmed from (10) that the MEC ($R_s$) thus derived satisfies the following:

$$\frac{\partial R_s}{\partial Y^e} > 0, \ \frac{\partial R_s}{\partial w} > 0, \ \frac{\partial R_s}{\partial K_s} < 0 \tag{12}$$

Therefore, in our model the MES changes when exogenous variables such as $Y^e$ or $w$ change[9].

Lastly, the effects of changes in exogenous variables on the firm's optimal investment plan are summarized as follows. An increase in the expected aggregate demand $Y^e$ at the initial time $s$ shifts the $\dot{I}_t = 0$ line in Figure 1 upward, causing instantaneous upward jumps in the MEC ($R_s$), the stock price ($q_s$) and the investment level ($I_s$) at time $s$, and then raises the capital stock in the steady state. An increase in the real wage $w$ also shifts the $\dot{I}_t = 0$ line upward and yields the same changes as an increase in $Y^e$. The reason why an increase in $w$ stimulates investment is that such a change lowers the relative price of capital and encourages factor substitution from labor to capital. Finally, an increase in the expected interest rate $r^e$ shifts the $\dot{I}_t = 0$ line downward, leading to an instantaneous drop in the stock price and the investment level at time $s$, subsequently reducing the capital stock in the steady state. This is because an increase

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homogenous in $I$ and $K$. In our framework, neither of these conditions hold.

[9] Nagatani (1981, ch 6) further clarified the difference between the schedule of the MEC (i.e., the relationship between $R$ and $K$ which satisfies (11)) and the schedule of the marginal efficiency of *investment* (i.e., the relationship between $r$ and $I$ which satisfies (7.a)), and stressed that the schedule which determines the current rate of investment is not the former but the latter.

in the expected interest rate shrinks the present discount value of the net cash flow stream and discourages the firm's investment activity.

## 3. "Short-run" macro-dynamics

In this section we construct a "Keynesian" macro-dynamic model and explore the properties of its "short-run" behavior. Here, the term "Keynesian" means that the equilibrium output at any time is determined by the principle of effective demand. The term "short-run" will be defined later.

Let us investigate macroeconomic equilibrium. Regarding the behavior of the firm, we suppose the followings. First, the firm finances the total investment cost (i.e., $I_s + c(I_s)$) from the household savings through the external financial market.[10] Second, the firm is assumed to distribute all sales to the household sector in the form of either the wage income ($wL_s$) or the profit ($\pi_s \equiv Y_s - wL_s$).

Regarding the household sector, we suppose that there are many homogenous households and their number is normalized to one without any loss of generality. Each household is endowed with constant units of labor $\bar{L}$ at each point in time, but the household supplies labor at a level which is equal to the firm's labor demand $L_s$ due to the existence of the demand constraint. Since the household is also an owner of the firm, it receives not only the wage income $wL_s$ as an employee, but also the firm's profit $\pi_s(\equiv Y_s - wL_s)$ as a stockholder at time $s$. Under this setting the household receives the total income equal to the firm's output level $Y_s$. For decisions regarding the household's consumption, we assume the following most simple Keynesian consumption function:

$$C_s = cY_s \quad (0 < c < 1) \tag{13}$$

where $c$ is the constant marginal (average) propensity to consume. Since the government and foreign sectors are neglected in our model, the actual aggregate demand

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[10] If we assume that the investment cost is internally financed (i.e., financed by the firm's retained earnings), the equilibrium output cannot be determined as usual through the principle of effective demand. Since the choice between internal and external finance does not have any impact on the investment plan itself under perfect financial market, this assumption is justifiable.

of the final product is

$$AD_s = C_s + I_s + c(I_s) = cY_s + I_s + c(I_s), \tag{14}$$

where the investment demand $I_s$ is derived from the firm's investment problem (5) in the previous section. Here, note that the adjustment cost of investment ($c(I_s)$) is assumed to take the form of expenditure of the final product. The equilibrium condition of the final product market is given by

$$Y_s = cY_s + I_s + c(I_s). \tag{15}$$

Therefore, the equilibrium output $Y_s^*$ at time $s$ can be derived as follows:

$$Y_s^* = s^{-1}[I_s + c(I_s)] \quad (s \equiv 1 - c) \tag{16}$$

where $s(\equiv 1 - c)$ is the marginal (average) propensity to save. From (16) we can confirm that $Y_s^*$ is a monotonically increasing function of $I_s$.

Note that under the "short-run" macroeconomic equilibrium derived in (16), the firm's subjective expectations $(Y^e, r^e)$ is not self-fulfilling (i.e., those expectations are not consistent with their realization values $(Y_s^*, r_s^*)$). In this paper we use the term "short-run" as the period during which such inconsistency remains. In other words, the term "short-run" means the situation where even if the firm's subjective expectations $(Y^e, r^e)$ deviate from their realization values $(Y_s^*, r_s^*)$, the firm does not correct the expectations.[11]

Since in the short-run the firm's expectations remain fixed, the optimal investment plan derived in (8.a), (8.b) and (8.c) becomes equal to the actual investment path. Once the investment path is determined, the output path $\{Y_t^*\}_{t=s}^{\infty}$ is also determined by (16). Therefore, the short-run macro-dynamics is essentially the same as one shown in Figure 1 in the previous section, and the effects of changes in exogenous variables on the short-run macro-dynamics are summarized as follows. First, an increase in the expected aggregate demand $Y^e$ or the fixed wage $w$ stimulates MEC, stock price, investment and output at that time, and raises output and capital stock in the steady state. Second, an increase in the expected interest rate $r^e$ discourages MEC, stock price, investment

[11] Note that the term "short-run" used in this paper is different from the usual meaning of the term. The same is true for the concepts of "long-run" defined later.

and output at that time, and lowers output and capital stock in the steady state. Finally, an increase in the saving rate $s$ does not have any direct impact on the firm's investment decision (i.e., it has no impact on MEC, stock price and investment at that time and capital stock in the steady-state), while such a change lowers the equilibrium output at any point in time through a decline in the aggregate demand.

## 4. Business cycle as a deviation from the "long-run" steady state

In this section we focus on the "long-run" behavior of the economy. In this paper the "long-run" is defined as the situation in which the firm's expectations $(Y^e, r^e)$ coincide with their realization values $(Y_S^*, r_S^*)$ (i.e., the expectations are self-fulfilling). From (8.a), (8.b) and (16), the short-run steady state is characterized as follows.

$$(r^e + \delta)[1 + c_I(I^*)] + wG_K(\delta^{-1}I^*, Y^e) = 0 \tag{17.a}$$

$$Y^* = s^{-1}[I^* + c(I^*)] \tag{17.b}$$

When the firm's expectations differ from their realization values in the short-run steady state shown by (17.a) and (17.b), the expectations will continue to be betrayed permanently. In such a situation any firm which is not irrational will correct its mistaken expectation. In this paper we adopt the following assumption regarding the firms' expectation revision behavior:

**Assumption**: *When the firm's expectations deviates from their realization values in the short-run steady state, the firm modifies the expectations toward their realization value.*

In other words, this assumption states that when the economy stays on the *transition process* to the short-run steady state, the firm does not modify the expectations even if they deviate from their realization value. This assumption is necessary in formulating Keynes's business cycle theory in its simplest form, though it is somewhat ad hoc. Following justifications may be possible for this assumption. First, the firm's capital investments such as building a factory, for example, take time[12], and it is not realistic to continuously revise expectations and change investment plans during the construction phase. In this sense, the above assumption is not so unrealistic. Second, this assumption may not be valid when the economy stays on the transition process for a fairly long period. In this paper, however, we focus on the case where the economy mainly stays in the long-run steady state, sometimes deviating from it for a while due to changing expectations. In such a case this assumption is not so problematic. It will be a future task to improve our model by adopting a more convincing expectation formation hypothesis. (See concluding remarks on this point.)

In this section, we further assume that only the expected aggregate demand $Y^e$ is adjusted in the short-run steady state (i.e., the expected interest rate $r^e$ is not adjusted and remains fixed) to simplify the discussion. The case where both $Y^e$ and $r^e$ are adjusted simultaneously will be discussed in the next section.

When the firm revises the expected aggregate demand $Y^e$ according to the above assumption, does an arbitrarily chosen expectation stably converge to a self-fulfilling one? From (17.a), the firm's investment in the short-run steady state $(I^*)$ can be expressed as

$$I^* = I(Y^e, r^e), \quad I_Y^* > 0, \quad I_r^* < 0, \tag{18}$$

where $I_Y^* \equiv \partial I^*/\partial Y^e$ and $I_r^* \equiv \partial I^*/\partial r^e$. Accordingly, from (17.b) and (18), the relation between the expected aggregate demand $Y^e$ and its realization value in the short-run steady state $Y^*$ is

$$Y^* = s^{-1}\left[I(Y^e, r^e) + c\big(I(Y^e, r^e)\big)\right], \quad Y_Y^* > 0, \quad Y_r^* < 0, \tag{19}$$

where $Y_Y^* \equiv \partial Y^*/\partial Y^e$ and $Y_r^* \equiv \partial Y^*/\partial r^e$.

In this paper, we assume that the function $Y^*(Y^e)$ is a strictly concave function as depicted in the Figure 2. To do so, the following three assumptions are necessary.

$$(r^e + \delta)[1 + c_I(0)] + wG_K(0,0) = 0 \tag{20.a}$$

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[12] According to empirical evidences, it is reasonable to estimate that the average length from start to completion of a new investment project is about 2 years. See Mayer (1960), Taylor (1982), Montogomery (1995), Peeters (1996) and Koeva (2000) for this point.

$$Y_Y^*|_{Y^e=0}\left(\equiv \frac{\partial Y^*}{\partial Y^e}\bigg|_{Y^e=0}\right) = s^{-1}\left[1 + c_I\big(I(0,r^e)\big)\right] I_Y^*(0,r^e) > 1 \tag{20.b}$$

$$Y_{YY}^*\left(\equiv \frac{\partial Y_Y^*}{\partial Y^e}\right) = s^{-1}[I_{YY}^*(1+c_I) + (I_Y^*)^2 c_{II}] < 0 \tag{20.c}$$

(20.a) means that the function $Y^*(Y^e)$ is a curve passing through the origin. (20.b) indicates that the slope of the function $Y^*(Y^e)$ at $Y^e = 0$ is greater than 1. (20.c) shows that the second-order derivative of $Y^*(Y^e)$ is negative.

(Figure 2 around here)

In order to consider these conditions more concretely, let us specify the production function and the investment adjustment cost function as follows.

$$F(K_t, L_t) = K_t^\alpha L_t^{1-\alpha}, \quad c(I_t) = \psi I_t^2/2 \tag{21}$$

In this case the equation (17.a) can be rewritten as

$$Y^e = A(1+\psi I^*)^{1-\alpha} I^*. \quad \left(A \equiv \delta^{-1}\left(\frac{(1-\alpha)(r^e+\delta)}{\alpha w}\right)^{1-\alpha}\right) \tag{22}$$

From (22) and (19) we can see that (20.a) holds under this specification. From (20.b), (21) and (22) we can derive

$$Y_Y^*|_{Y^e=0}\left(\equiv \frac{\partial Y^*}{\partial Y^e}\bigg|_{Y^e=0}\right) = \frac{\delta}{s}\left(\frac{(1-\alpha)(r^e+\delta)}{\alpha w}\right)^{-(1-\alpha)}. \tag{23}$$

Therefore, under the specification (21), the assumption (20.b) is equivalent to

$$\frac{\delta}{s}\left(\frac{(1-\alpha)(r^e+\delta)}{\alpha w}\right)^{-(1-\alpha)} > 1. \tag{24}$$

Concerning (20.c), from (22) we can show

$$I_Y^* > 0, \quad I_{YY}^* < 0. \tag{25}$$

However, even taking this result into account, the sign of $Y_{YY}^*$ in (20.c) is still ambiguous. Therefore, even under the specification (21), (20.c) needs to be assumed.

If the function $Y^*(Y^e)$ is as shown in Figure 2, then the expectation modification process under the Assumption above is globally stable, so any short-run steady state converges to the unique long-run steady state $Y^{e*}$ even if the initial value of $Y^e$ is chosen arbitrarily.

Based on the expectation revision process described so far, let us examine the mechanism of business cycles driven by changes in the expectation. Consider the situation in which the economy initially has been staying in a long-run steady state ($E_0(Y_0^*, K_0^*)$ in Figure 4 below) where the expectation is self-fulfilling (i.e., $Y^e = Y_0^*$). When some change in the firm's expectation has occurred, how does the economy respond to such an exogenous expectation shock?

Figure 3.a and Figure 3.b illustrate a series of changes in the economy when the firm's expected aggregate demand suddenly rise to a level ($Y_1^e$) which is higher than the self-fulfilling one ($Y^e = Y_0^*$). Figure 3.a depicts the change in the optimal investment path when expectations change. Figure 3.b depicts the relationship between the expectation and its realization value (i.e., equilibrium output in the short-run steady state).

(Figure 3.a and Figure 3.b around here)

First of all, when the firm adopts the new expectation $Y_1^e$, the $\dot{I}_t = 0$ line shifts upward (i.e., the change ↑(a) in Figure 3.a and 3.b), and the economy moves from the initial long-run steady state $E_0$ to the new short-run steady state $E_1(Y_1^*, K_1^*)$. As is obvious from the argument in the previous section, during this process the economy experiences a "boom" because MEC, stock price, investment and equilibrium output instantaneously jump, and equilibrium output and capital stock in the new short-run steady state ($E_1$) also rise.

However, the economy cannot continue to stay in $E_1$, because it can be confirmed from Figure 3.b that the firm's expectation $Y_1^e$ exceeds its realization value $Y_1^*$ in $E_1$. Accordingly, the firm corrects the expectation downward and adopts a new expectation, for example, $Y_2^e$ which is smaller than $Y_1^e$. This change shifts the $\dot{I}_t = 0$ line downward (i.e., the change ↓(b) in Figure 3.a and 3.b), and the economy moves from $E_1$ to the next short-run steady state $E_2(Y_2^*, K_2^*)$. During this transition process the economy experiences a "slump" when MEC, stock price, investment and equilibrium output instantaneously fall, and equilibrium output and capital stock in the new steady state ($E_2$) also decline relative to those in $E_1$. During this process the level of capital depreciation ($\delta K_s$) exceeds the gross investment level ($I_s$) and the capital stock declines continuously. Therefore, when the capital depreciation rate becomes larger, the over-accumulated capital induced by the firm's optimistic demand expectation depreciates more rapidly, and as a result the period of slump that the economy experiences becomes shorter.

In the case where $Y_2^* > Y_0^*$ holds, the firm must still revise its expectation downward because it is still higher than its realization value in the steady state, and the economy will eventually converge to the initial long-run steady state $E_0$. On the other hand, if the firm modifies the expectation to an excessively low level such that $Y_2^e < Y_0^*$, the extent of the downward shift of the $\dot{I}_t = 0$ line becomes fairly large (i.e., the change ↓(c) in Figure 3.a and 3.b), and the economy experiences a more severe slump (namely, a "depression") during the transition from $E_1$ to $E_3$. Such a deep slump is often called a "negative bubble", because in such a situation the firm's revised expectation is too pessimistic in that it is lower even than the self-fulfilling expectation in the long-run steady state $E_0$, causing an excessive drop in the stock price. However, the economy recovers from this severe slump sooner or later, returning to the initial long-run steady state $E_0$, because the firm's expectation is lower than its realization value in $E_3$ and the firm accordingly revises the expectation upward.

It is clear that a business cycle model formulated in this manner corresponds exactly to Keynes' view of business cycles already stated at the beginning of the introduction. It is remarkable that this view of Keynes regarding the business cycle cannot be formulated if we assume the "perfect foresight" of the firm. In fact, if such an assumption is imposed, the economy always stays in the long-run steady state and as a consequence the business cycle cannot be described as being governed by optimistic and pessimistic surges in expectations. In this sense our model is different from, for example, Farmer (2016) which attempts to formulate Keynes' "animal spirit" using the concept of indeterminacy and sunspot shock under the mainstream macroeconomic theory.

## 5. Analysis of the case where both are $Y^e$ and $r^e$ adjusted

In the previous section we examined the case where only the expected aggregate demand $Y^e$ is adjusted and the expected interest rate $r^e$ remains fixed. In this section we investigate the case where both $Y^e$ and $r^e$ are adjusted.

The actual interest rate at time $s$ can be defined as $r_s = (Y_s - wL_s)/K_s$, so from (17.b) and $I^* = \delta K^*$, the actual interest rate in the short-run steady state can be written as

$$r^* = \frac{Y^* - wG(K^*, Y^e)}{K^*} = \frac{\delta}{s}\left(1 + \frac{c(I^*)}{I^*}\right) - \delta w \frac{G(\delta^{-1}I^*, Y^e)}{I^*}. \tag{26}$$

In the following analysis, we specify the production function and the investment adjustment cost function as in (21). In this case we have

$$r^*(Y^e, r^e) = \frac{\delta}{s}\left(1 + \frac{1}{2}\psi I^*(Y^e, r^e)\right) - \frac{1-\alpha}{\alpha}(r^e + \delta)[1 + \psi I^*(Y^e, r^e)]. \tag{27}$$

Here, we make the following assumption.

$$\frac{\delta}{2s} - \frac{1-\alpha}{\alpha}(r^e + \delta) > 0 \tag{28}$$

Under this assumption we can easily confirm

$$r_Y^* > 0, \quad r_r^* < 0, \tag{29}$$

where $r_Y^* \equiv \partial r^*/\partial Y^e$ and $r_r^* \equiv \partial r^*/\partial r^e$. $r_Y^* > 0$ means that a rise in the expected aggregate demand $Y^e$, which increases output (aggregate income) in the short-run equilibrium (see (19)), raises the capital income (i.e., the profit of the firm). Similarly, $r_r^* < 0$ means that a rise in the expected interest rate $r^e$, which reduces output (aggregate income) in the short-run equilibrium (see (19)), lowers the capital income.

As in the previous section, we assume that if the firm's expectations $(Y^e, r^e)$ deviate from their realized values in the short-run steady state, the firm adjusts the expectations in response to their realized values. The expectation revision process therefore can be formulated as follows.

$$\dot{Y}_e = \beta_1[Y^*(Y^e, r^e) - Y^e] \quad (\beta_1 > 0) \tag{30.a}$$

$$\dot{r}_e = \beta_2[r^*(Y^e, r^e) - r^e] \quad (\beta_2 > 0) \tag{30.b}$$

As is proved in the Appendix 1, there exists the unique steady state $(Y^{e*}, r^{e*})$ in this dynamic system. Figure 4 depicts the phase diagram of this system. As can be seen from this figure, the steady state is stable. (See Appendix 2 for a mathematical proof of local stability). Therefore, even if the initial values of $(Y^e, r^e)$ are chosen arbitrarily, the economy eventually converges to this long-run steady state where both expectations are self-fulfilling. In the following, we explain how the economy evolves when the expected aggregate demand $Y^e$ suddenly rises in the long-run steady state.

(Figure 4 around here)

Suppose that the economy was initially located at $E_0$ in Figure 4. We denote the firm's expectations at $E_0$ as $(Y_0^e, r_0^e)$. Since $E_0$ is the long-run steady state, $Y_0^e = Y^{e*}$ and $r_0^e = r^{e*}$ holds. Suppose that $Y_0^e$ suddenly rises while $r_0^e$ remains unchanged (i.e., $Y_1^e > Y_0^e$ and $r_1^e = r_0^e$). Then, the economy jumps from $E_0$ to $E_1$. We denote the realized output and interest rate in the short-run steady state under these new expectations $(Y_1^e, r_1^e)$ as $(Y_1^*, r_1^*)$. Recall that the followings hold (see (19) and (29)).

$$Y_Y^* > 0, \quad Y_r^* < 0, \quad r_Y^* > 0, \quad r_r^* < 0. \tag{31}$$

Considering (31), we can confirm $Y_1^*(Y_1^e, r_1^e) > Y_0^*(Y_0^e, r_0^e)$ and $r_1^*(Y_1^e, r_1^e) > r_0^*(Y_0^e, r_0^e)$, where $Y_1^*(Y_1^e, r_1^e)$ and $r_1^*(Y_1^e, r_1^e)$ are the realized output and interest rate in $E_1$. Namely, when the economy jumps from $E_0$ to $E_1$, the realized output and interest rate in the new steady state become higher. Since the realized output is an increasing function of the investment level $I_t$ (see (17.b)) and $I_t$ is an increasing function of the "stock price" $q_t$ (see (7.a)), the investment level and the stock price in the short-run steady state of $E_1$ are also higher than those of $E_0$. On the other hand, how the MEC (the marginal efficiency of capital defined in (11)) changes when the economy jumps from $E_0$ to $E_1$ is ambiguous. This is because the MEC increases as $Y^e$ increases and decreases as $K_t$ increases (see (12)), and both $Y^e$ and $K_t$ in the short-run steady state of $E_1$ are higher than those of $E_0$.

As can be seen from the directions of arrows in Figure 4, at $E_1$ the expected aggregate demand $Y_1^e$ is revised downward while the expected interest rate $r_1^e$ is revised upward, which implies that $Y_1^* < Y_1^e$ and $r_1^* > r_1^e$ hold. As a result, the economy will eventually reach $E_2$. Taking (31) into account, we can easily confirm $Y_2^*(Y_2^e, r_2^e) < Y_1^*(Y_1^e, r_1^e)$ and

$r_2^*(Y_2^e, r_2^e) < r_1^*(Y_1^e, r_1^e)$. Namely, when the economy moves from $E_1$ to $E_2$, the realized output and interest rate in the short-run steady state become lower. Furthermore, since the realized output declines during this process, the investment level and the stock price also fall.

From the directions of arrows in Figure 4, both $Y_2^e$ and $r_2^e$ are revised downward at $E_2$, which implies that $Y_2^* < Y_2^e$ and $r_2^* < r_2^e$ hold. As a result, the economy will eventually reach $E_3$. It is ambiguous how the realized output and interest rate will change when the economy moves from $E_2$ to $E_3$. Namely, it is not clear which is larger, $Y_3^*(Y_3^e, r_3^e)$ or $Y_2^*(Y_2^e, r_2^e)$, (and $r_3^*(Y_3^e, r_3^e)$ or $r_2^*(Y_2^e, r_2^e)$). This is because a downward revision of $Y^e$ has negative impacts on both the realized output and interest rate, while a downward revision of $r^e$ has positive impacts on them. Since the change in the realized output is ambiguous, the changes in the investment level and the stock price from $E_2$ to $E_3$ are also ambiguous.

Subsequent changes in the economy can be discussed in the same way. The results are summarized in Table 1. Changes in the investment level and the stock price are omitted because they move in the same direction as the realized output. Changes in the MEC are also omitted because they are always ambiguous.

(Table 1 around here)

## 6. Concluding remarks

In this paper we have constructed a simple model that closely reflects Keynes' own view of business cycle which is stated in Chapter 22 of *General Theory*. Our model is traditional in that the equilibrium output is determined through the principle of effective demand, but the main difference from the standard post Keynesian models is to explicitly introduce the intertemporal optimization of the firm's investment decision under subjective expectations. Using this framework, we demonstrated that Keynes' view of business cycles can be reproduced as a deviation from the long-run steady state where the expectations are self-fulfilling.

In this paper we made a very simplifying assumption regarding the expectation formation that the firm adaptively adjusts the expectations in response to their realized value only in the short-run steady state. While this assumption is necessary in formulating Keynes's view in its simplest form, this assumption is rather ad hoc. Keynes himself thought that booms are caused by the over-investment which is brought about by the optimistic expectations that are destined to disappointment, and that the economy will enter a phase of recession after the sudden collapse of such optimistic expectations. As pointed out in footnotes 1 and 5, attempts to formulate boom and bust cycles driven by waves of optimism and pessimism have been vigorously developed by Branch and McGough (2010), Branch and Evans (2011), De Grauwe (2011, 2012), Franke (2012, 2014), Flaschel et.al (2018) and so on. In these studies there are two types of agents (i.e., one who forms forward-looking expectation and the other backward-looking expectation) and the ratio of these two types changes endogenously. While it seems difficult to regard these models as a faithful formulation of Keynes's business cycle theory, these models are superior to our model in that they formulate the waves of optimism and pessimism based on the findings of behavioral economics and construct models of endogenous business cycles. It will be a future task to elaborate our model by referring to such formulations of expectation formation.

## Appendix 1: Proof of uniqueness of the long-run steady state in expectation dynamics

In this appendix we show that there exists the unique long-run steady state for expectation dynamics given by (30.a) and (30.b). In the long-run steady state we have the followings.

$$\left(\dot{Y}_e = 0\right) \quad Y^*(Y^e, r^e) = Y^e \tag{A.1.a}$$

$$(\dot{r}_e = 0) \quad r^*(Y^e, r^e) = r^e \tag{A.1.b}$$

First, we can easily show

$$\left(\dot{Y}_e = 0\right) \quad \frac{dr^e}{dY^e} = \frac{1 - Y_Y^*}{Y_r^*} < 0, \tag{A.2.a}$$

$$(\dot{r}_e = 0) \quad \frac{dr^e}{dY^e} = \frac{r_Y^*}{1 - r_r^*} > 0, \tag{A.2.b}$$

where we use the fact that under the assumptions (20.b) (or (24)) and (20.c) the value of $Y_Y^*$ evaluated with $(Y^e, r^e)$ that satisfies $\dot{Y}_e = 0$ satisfies $Y_Y^* < 1$ (or $1 - Y_Y^* > 0$). (A.2.a) (resp. (A.2.b)) means that the $\dot{Y}_e = 0$ (resp. $\dot{r}_e = 0$) line is an downward (resp. upward) sloping curve in the $(Y^e, r^e)$ plane.

Next, let us examine the values of $Y^e$ which satisfy $Y^*(Y^e, \infty) = Y^e$ and $Y^*(Y^e, 0) = Y^e$ in turn, where $Y^*(Y^e, \infty) \equiv lim_{r^e \to \infty} Y^*(Y^e, r^e)$ and $Y^*(Y^e, 0) \equiv lim_{r^e \to 0} Y^*(Y^e, r^e)$.

From (22), $I^*(Y^e, \infty)\left(\equiv lim_{r^e \to \infty} I^*(Y^e, r^e)\right) = 0$ holds. From (19) and (21), $Y^*(Y^e, r^e)$ is given by $Y^*(Y^e, r^e) = s^{-1}[I^*(Y^e, r^e) + 2^{-1}\psi\{I^*(Y^e, r^e)\}^2]$. Hence, $Y^*(Y^e, \infty) = 0$ also holds. In sum, when $r^e$ approaches to infinity, the value of $Y^e$ which satisfies $Y^*(Y^e, \infty) = Y^e$ approaches to zero.

On the other hand, when $r^e$ approaches to zero, the coefficient $A$ in (22) approaches to some finite value $\bar{A}\left(= \delta^{-\alpha}[(1-\alpha)/\alpha]^{1-\alpha} w^{-(1-\alpha)}\right)$, so $I^*(Y^e, 0)\left(\equiv lim_{r^e \to 0} I^*(Y^e, r^e)\right)$ can be depicted as follows.

(Figure 5 around here)

Accordingly, under the assumptions (20.b) (or (24)) and (20.c), $Y^*(Y^e, 0)$ can also be depicted by Figure 2. Therefore, the value of $Y^e$ which satisfies $Y^*(Y^e, 0) = Y^e$ will be some finite value.

Finally, let us examine the values of $r^e$ which satisfies $r^*(0, r^e) = r^e$, where $r^*(0, r^e) \equiv lim_{Y^e \to 0} r^*(Y^e, r^e)$. From (22), $I^*(0, r^e)\left(\equiv lim_{Y^e \to 0} I^*(Y^e, r^e)\right) = 0$ holds. From (27), $r^*(0, r^e)$ can be calculated as

$$r^*(0, r^e) = \frac{\delta}{s} - \frac{1-\alpha}{\alpha}(r^e + \delta). \tag{A.3}$$

Hence the value of $r^e$ which satisfies $r^*(0, r^e) = r^e$ approaches to

$$r^e = \alpha\delta\left(\frac{1}{s} - \frac{1-\alpha}{\alpha}\right). \tag{A.4}$$

We can confirm that the sign of $r^e$ in (A.4) is positive under the assumption (28).

From the above discussion, both the $\dot{Y}_e = 0$ line and the $\dot{r}_e = 0$ line can be depicted as shown in Figure 4, and there exists the unique long-run steady state for the expectation dynamics.

**Appendix 2: Proof of local stability of expectation dynamics**

In this appendix we show the local stability of expectation dynamics given by (30.a) and (30.b). By linearly approximating (30.a) and (30.b) in the neighborhood of the long-run steady state $(Y^{e*}, r^{e*})$, we have

$$\begin{pmatrix} \dot{Y^e} \\ \dot{r^e} \end{pmatrix} = B \begin{pmatrix} Y^e - Y^{e*} \\ r^e - r^{e*} \end{pmatrix}. \quad \left( B \equiv \begin{bmatrix} \beta_1 (Y_Y^* - 1) & \beta_1 Y_r^* \\ \beta_2 r_Y^* & \beta_2 (r_r^* - 1) \end{bmatrix} \right) \tag{B.1}$$

The signs of trace and determinant of the coefficient matrix $B$ are as follows.

$$tr(B) = \beta_1 (Y_Y^* - 1) + \beta_2 (r_r^* - 1) < 0 \tag{B.2.a}$$

$$\det(B) = \beta_1 \beta_2 [(Y_Y^* - 1)(r_r^* - 1) - Y_r^* r_Y^*] > 0 \tag{B.2.b}$$

Hence, two eigenvalues of the coefficient matrix $B$ are both negative, which means that the expectation dynamics is locally stable.

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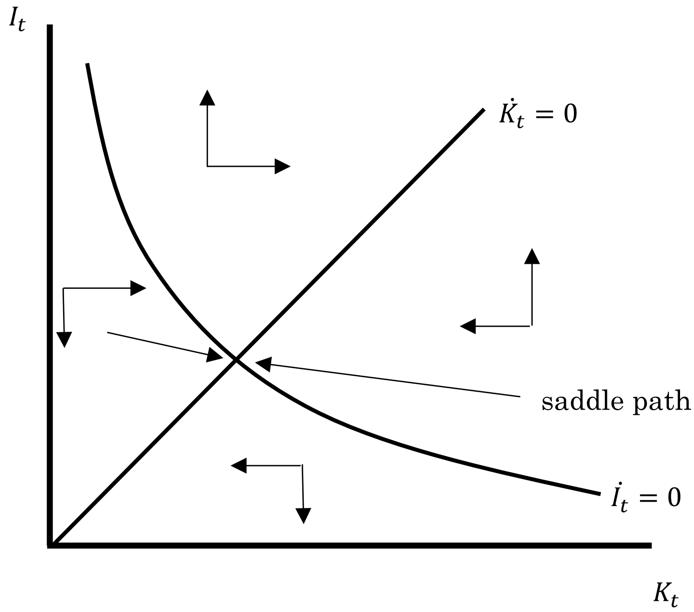
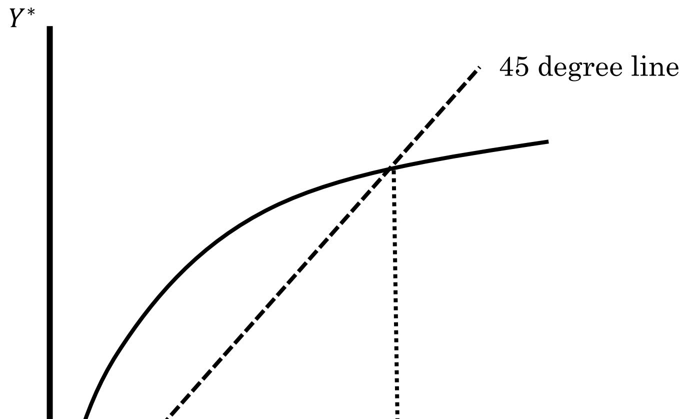
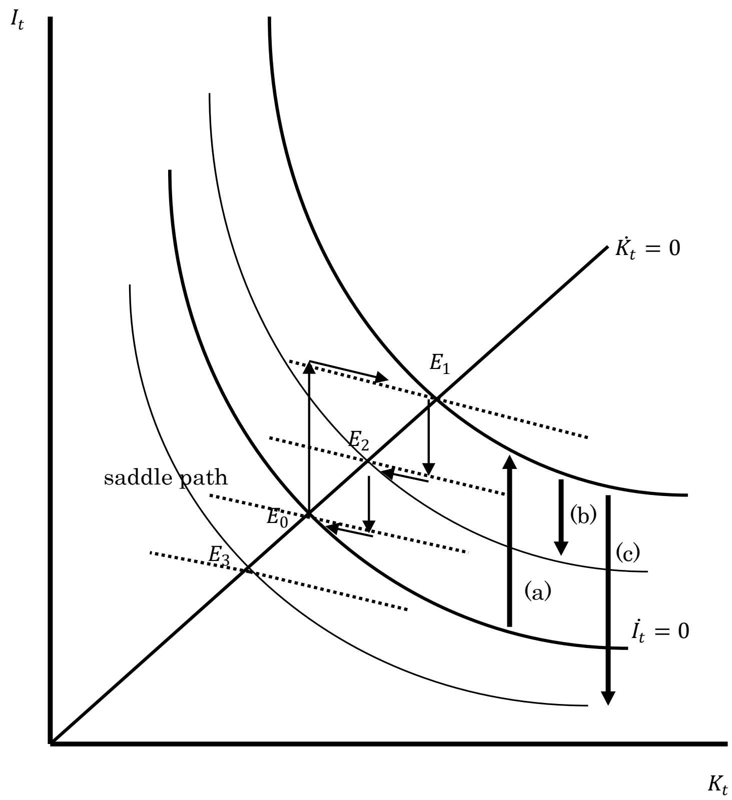



(Figure 1: The optimal investment path)

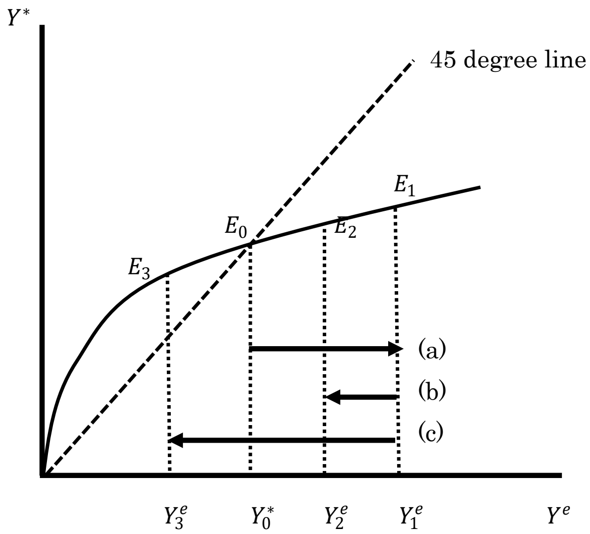


$Y^{e*}$ $Y^e$

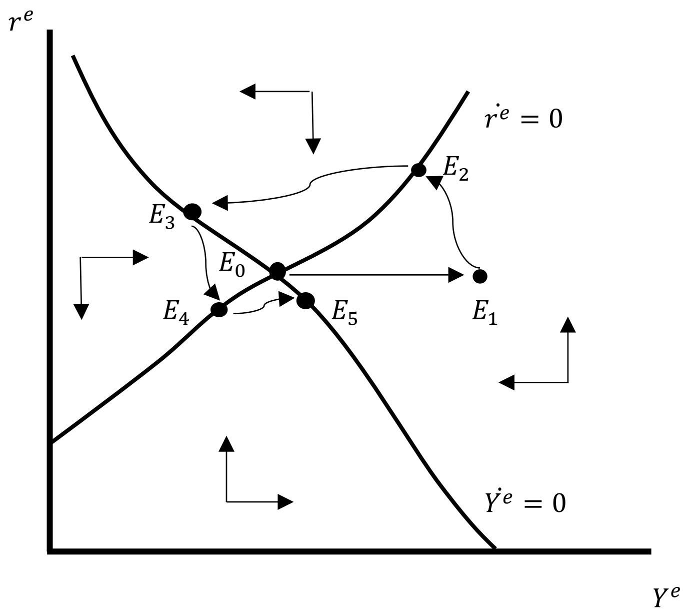
(Figure 2: Figure of function (19))



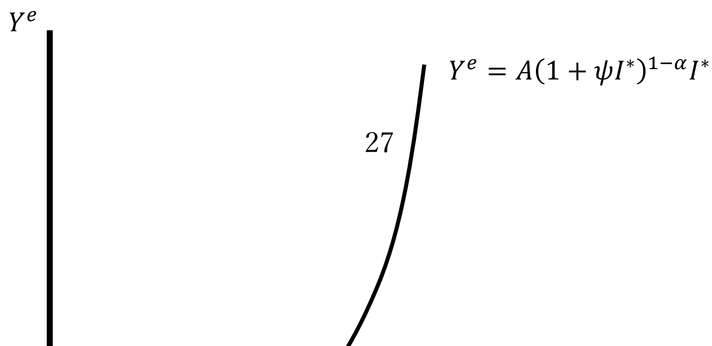

(Figure 3.a : Business cycle as a deviation from the long-run steady state)

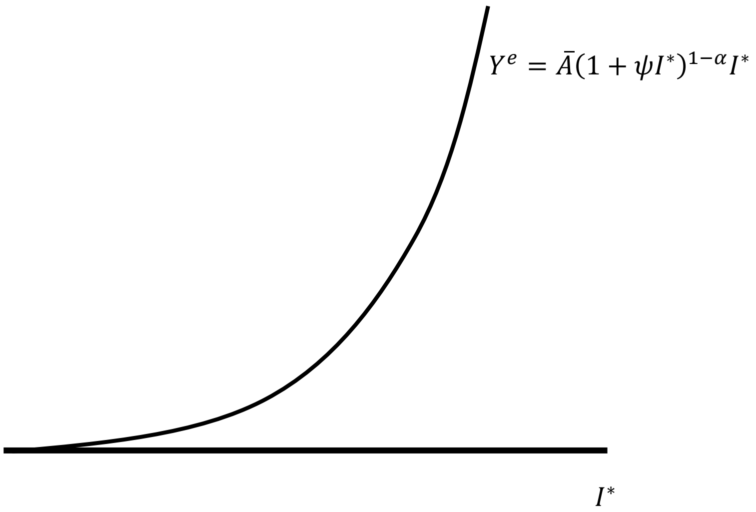



(Figure 3.b : Business cycle as a deviation from the long-run steady state)




(Figure 4: Phase diagram of (30.a) and (30.b))






(Figure 5: Figure of function $I^*(Y^e, 0)$)

| | $E_0 \to E_1$ | $E_1 \to E_2$ | $E_2 \to E_3$ | $E_3 \to E_3$ | $E_4 \to E_5$ |
|---|---|---|---|---|---|
| $Y^*$ | + | − | ? | + | ? |
| $r^*$ | + | − | ? | + | ? |

(Table 1: The effects of a sudden rise in $Y^e$ in the dynamic system of (30.a) and (30.b))