The Society for Economic Studies
The University of Kitakyushu
Working Paper Series
No.2020-4(accepted in March 29, 2021, revised in November 19, 2021)

Limited asset market participation and fiscal sustainability

Noritaka Maebayashi* Jumpei Tanaka†

Abstract

We examine the implications of limited stock market participation (the stylized fact that only a fraction of households directly own stocks) for economic growth and, in particular, fiscal sustainability. Constructing an overlapping generations (OG) model where (1) individuals can choose between two types of savings (i.e., physical capital with high returns but costly to hold and bank deposits with low returns but no costs), and (2) banks invest part of their total deposits in physical capital and use the rest to underwrite government debt, we show the following. First, an increase in the average level of an individual's financial literacy worsens fiscal sustainability while promoting economic growth under plausible parameter values. Second, if banks decrease (resp. increase) the number of government bonds they underwrite, fiscal sustainability improves (resp. worsens) when the average financial literacy of individuals is relatively low.

JEL classification: E13; G11; H63

Keywords: limited asset market participation, financial literacy, fiscal sustainability, public debt, overlapping generations, endogenous growth

*Faculty of Economics and Business Administration, The University of Kitakyushu, 4-2-1 Kitagata, Kokura Minami-ku, Kitakyushu, Fukuoka 802-8577, JAPAN; e-mail: non818mn@kitakyu-u.ac.jp. This research was supported by grants-in-aid from the Ministry of Education, Culture, Sports, Science, and Technology of the Japanese government, KAKENHI (grant number 20K13455).

†Faculty of Economics and Business Administration, The University of Kitakyushu, 4-2-1 Kitagata, Kokura Minami-ku, Kitakyushu, Fukuoka 802-8577, JAPAN; e-mail: j-tanaka@kitakyu-u.ac.jp

1 Introduction

Accumulated government debt is one of the most important problems common to all advanced countries. Figure 1 shows the evolution of the ratio of government debt to GDP for the G7 countries over the last 20 years. The figure shows that the ratio of government debt to GDP in G7 countries, with the exception of Germany, has been on an upward trend, and that among others, Japan's ratio is remarkably high. Motivated by this current situation, a number of theoretical and empirical studies on fiscal sustainability have been conducted. This study is one such attempt. Specifically, it examines how a stylized fact called "limited asset market participation" is related to fiscal sustainability (or the maximum sustainable level of government debt).

[Figure 1]

It is well-known that in developed countries, only a fraction of households own stocks, i.e., their participation in stock markets is limited (Mankiw and Zeldes (1991), Haliassos and Bertaut (1995), Campbell (2006), and so on).[1] Various reasons can be adduced as to why stock market participation is limited,[2] but the relationship between financial literacy (the ability to properly gather and analyze investment information) and stock market participation is one reason that has received special attention recently. Holding risky assets, such as stocks, requires a higher level of financial literacy than holding safe assets, such as bank deposits. Since acquiring financial literacy comes at a cost, it is reasonable to forgo participation in the stock market if its cost is high. In fact, many empirical studies, such as Kimball and Shumway (2006), van Rooij, Lusardi, and Alessie (2011), Yoong (2011), and Thomas and Spataro (2018) show that low financial literacy significantly reduces the rate of stock market participation.[3]

What impact would the above characteristics of limited stock market participation have on fiscal sustainability (or the maximum sustainable level of government debt)? Consider a situation where participation in the stock market is stimulated by the increased financial literacy of

[1]Thomas and Spataro (2018) reports that the average stock market participation rate of households for 9 European countries in year 2010 was 16.84%. Fujiki, Hirakata, and Shioji (2012) confirm that in Japan at most 15% of households held stocks for the period from 2007 to 2010. See also Guiso, Haliassos, and Jappelli (2003) and Christelis, Jappelli and Padula (2010) for other data.

[2]Haliassos and Bertaut (1995) point to short-sale constraints, income risk, inertia, and departures from expected utility maximization as reasons for limited participation. Constantinides et al. (2002) argue for the liquidity constraints young people face. Besides these factors, sociological explanations such as trust and culture (Guiso et al. 2008) and the influence of neighbors and peers (Hong et al. 2004; Brown et al. 2008) have also been offered.

[3]See also Guiso and Jappelli (2005) and Christelis, Jappelli and Padula (2010). The former (resp. the latter) investigate the relationship between financial awareness (resp. cognitive abilities) and stock market participation.

individuals. This situation, on the one hand, makes it easier for firms to raise capital investment funds through the stock market, and this could have a positive impact on economic growth. Since the ratio of government debt to GDP gradually decreases with increased economic growth, it improves fiscal sustainability. Increases in financial literacy and stock market participation, on the other hand, will reduce the share of individuals who hold safe assets such as bank deposits. Table 1 shows the proportion of domestic outstanding government bonds held by domestic and foreign banks in G7 and several other countries in 2020. This table suggests that private banks are non-negligible holders of government bonds, and accordingly that savings in the form of bank deposits are more likely to be used to underwrite government debt. Hence, increases in financial literacy and stock market participation could worsen fiscal sustainability by reducing the underwriters of government bonds and pulling up the government bond yield. Which of these two conflicting effects dominates? The purpose of this study is to examine this point theoretically.

[Table 1]

To investigate the above issues, we construct a simple overlapping generations (OG) model with the following characteristics:

- There are three types of assets: physical capital (which corresponds to stocks), bank deposits, and government bonds.
- When saving, individuals can choose between physical capital, which is more profitable but costly to hold, and bank deposits, which are less profitable but not costly to hold.
- In investing in physical capital, individuals must allocate a portion of their endowed time to acquiring financial knowledge. The more time they spend on it, the higher the return from their savings. Learning ability (more precisely, the extent to which time spent on improving financial literacy increases the return on savings) varies across individuals.
- Banks are assumed to invest a given proportion of their total bank deposits in physical capital and the rest in government bonds.
- The government manages its finances in such a way that past debts are repaid only by issuing new bonds (i.e., the government plays the "Ponzi game").

Under the above model setup, we show the following results.

First, there exists one stable balanced growth path (BGP) and one unstable BGP in our model. In the stable BGP, all individuals are physical capital holders; hence, the ratio of government debt to physical capital in the economy is zero. Contrariwise, the ratio of government debt to physical capital in the unstable BGP corresponds to its maximum sustainable value; if the actual ratio of government debt to physical capital in the initial period is larger than this value, the actual ratio will diverge and the government will go bankrupt. If the initial actual ratio is smaller than this value, on the other hand, the actual ratio will decline over time, and the economy eventually converges to the stable BGP.

Second, under plausible numerical settings of parameters, an increase in each individual's learning ability (or equivalently, each individual's financial literacy) reduces the maximum sustainable ratio of government debt to physical capital, while increasing the economic growth rates in both stable and unstable BGPs. This result is very important for countries that have introduced educational programs to improve financial literacy, with the increasing importance of financial literacy due to the increasing complexity of financial markets and rising life expectancy.[4] Our results imply that while such a policy certainly has the positive effect of raising the potential rate of economic growth, it also has the negative effect of worsening fiscal sustainability and increasing the likelihood of government bankruptcy.

Third, we examine the impact on fiscal sustainability when banks change their asset investment behavior. When banks decrease (resp. increase) the number of government bonds they underwrite, it would intuitively seem to worsen (resp. improve) fiscal sustainability. However, we show that when the average financial literacy of individuals is relatively low, the opposite result holds, that is, fiscal sustainability improves (resp. worsens) if banks decrease (resp. increase) the amount of government bonds they underwrite. This result implies that if banks decrease (resp. increase) the amount of government bonds they underwrite, the need for fiscal consolidation will conversely decrease (resp. increase) when the average financial literacy of individuals is relatively low.

[4] OECD (2014, p. 28) provides a brief overview of financial education efforts in such countries as New Zealand, Australia, Belgium, Shanghai-China, Czech Republic, Estonia, France, U.S., Colombia, Italy, Israel, and Spain.

Overview of previous literature

(A) Theoretical studies of fiscal sustainability

Previous studies that have investigated fiscal sustainability theoretically can be divided into two main categories: those based on the Ramsey model and those based on the overlapping generations model. The latter is relevant to this study.

Tirole (1985) examined the conditions for the existence of asset price bubbles using Diamond's (1965) neoclassical growth model and showed that positive bubbles can exist when the economy is dynamically inefficient. This result can also be interpreted as the sustainability condition of the government's Ponzi game. King (1992) and Grossman and Yanagawa (1993) reexamined the argument of Tirole (1985) using an AK-type endogenous growth model and showed that the government Ponzi games can be sustainable when the economic growth rate exceeds the private rate of return on physical capital, even if the economy is dynamically efficient. Chalk (2000) studied the sustainability of fiscal management with permanent primary deficit using Diamond's (1965) neoclassical growth model, and showed that it can be sustainable if the size of the permanent primary deficit is small. Bräuninger (2005) revisited Chalk's (2000) argument using an AK-type endogenous growth model and confirmed that a similar conclusion holds. Using an endogenous growth model with the public investment of Barro (1990), Arai (2011) showed that when public investment/GDP is small, an increase in public investment spending improves fiscal sustainability. Yakita (2008), using an endogenous growth model with public capital accumulation of Futagami et al. (1993) type, showed that a larger initial value of public capital raises the maximum sustainable level of government debt. Using a model of human capital accumulation through both private and public education, Motoyama (2019) showed that the maximum sustainable level of government debt does not depend on the initial value of human capital. Using a model with involuntary unemployment based on the efficient wage hypothesis, Yakita (2014) showed that an increase in the primary deficit raises unemployment and the amount of capital stock in the new steady state, while the impact on the maximum sustainable public debt/GDP is ambiguous. While all of the above arguments are based on a closed economy model, Farmer and Zotti (2010), based on a two-country, two-goods model, showed that there is a negative correlation between the maximum sustainable levels of public debt in both countries, and that raising the amount of public debt in one country worsens capital accumulation in both countries.

Thus, previous studies have considered the issue of fiscal sustainability in overlapping generations models under various model settings, but to the best of our knowledge, no study has investigated the relationship between limited asset market participation and fiscal sustainability. The contribution of this study is to derive new insights on this point.

(B) Theoretical studies of limited asset market participation

We now provide a brief overview of previous theoretical studies on limited asset market participation. Initially, the fact of limited asset market participation attracted attention on the grounds that asset price behaviors, which are difficult to understand rationally, can be explained to some extent by taking this fact into account (e.g., Mankiw and Zeldes,1991; Allen and Gale,1994). However, many recent studies of limited asset market participation have addressed this issue in relation to the theory of monetary policy. For example, Christiano and Eichenbaum (1995) and Alvarez et al. (2001) showed that the ”liquidity effect” of monetary policy (i.e., the phenomenon that monetary easing lowers the nominal interest rate), which cannot be explained in the complete market model, can be explained by assuming limited participation in asset markets. Gali et al. (2004) and Bilbiie (2008) point out that under limited asset market participation, local determinacy may not hold in the New Keynesian model, even if the Taylor principle[5] is assumed.

Thus, most of the theoretical studies of limited asset market participation are related to asset pricing theory or monetary policy theory, and to the best of our knowledge, no studies have considered this issue in the context of long-run economic growth.

Organization of this paper.

The remainder of this paper is organized as follows. We set up the model in Section 2 and examine the existence and stability of balanced growth paths (BGPs) in Section 3. Effects of changes in financial literacy (resp. banks’ asset investment behavior) on economic growth and fiscal sustainability are investigated in Section 4 (resp. 5). Finally, we conclude the paper in section 6.

[5]The Taylor principle means that when the inflation rate increases, the monetary authority increases the short-term nominal interest rate more than the inflation rate.

2 Setting up the model

In this section, we describe the setup of the model. Our model is an OG model consisting of four economic agents (individuals, firms, banks, and the government) and three assets (physical capital, bank deposits, and government bonds). It is basically an extension of Grossman and Yanagawa (1993) on the following three points: (1) Individuals can choose between physical capital and bank deposits when saving, (2) in investing in physical capital, individuals need to spend part of their endowed time acquiring financial knowledge, and (3) banks invest a given proportion of their total deposits in physical capital and the rest in government bonds.

2.1 Firms

The production sector is composed of homogenous firms, the number of which is normalized to 1. Each firm produces final goods using two kinds of production factors (i.e., physical capital and labor). We assume that the final goods market and factor markets are perfectly competitive, and the price of the final goods is normalized to 1 (i.e., the final goods are numeraire). The production function of each firm is given by

$$Y_t = F[K_t, A(\bar{K}_t)L_t]. \quad (\text{where} \quad A(\bar{K}_t) \equiv \theta\bar{K}_t).$$

Here, Y_t, K_t, and L_t are the outputs of the final goods, physical capital input, and labor input, respectively. We assume that the production function F exhibits constant returns to scale and positive marginal products with respect to each input. $A(\bar{K}_t)(\equiv \theta\bar{K}_t)$ is the labor-augmenting productivity, and it is an increasing function of $\bar{K}_t$ (i.e., average physical capital stock per the young generation's population). Following Romer (1986), we suppose that each firm maximizes its profit, considering $A(\bar{K}_t)$ as given. The first-order conditions of profit maximization are as follows:

$$F_K[K_t^d, \theta\bar{K}_t L_t^d] = r_t^k + \delta \quad (\text{where} \quad F_K \equiv \partial F/\partial K_t). \tag{1a}$$

$$\theta\bar{K}_t F_{AL}[K_t^d, \theta\bar{K}_t L_t^d] = w_t \quad (\text{where} \quad F_{AL} \equiv \partial F/\partial(\theta\bar{K}_t L_t)). \tag{1b}$$

Here, K_t^d, L_t^d, r_t^k, δ, and w_t are the demand for physical capital, demand for labor, (net) rate of return on physical capital, depreciation rate of physical capital, and wage, respectively.

2.2 Individuals

Each individual lives for two periods (young and old). The set of individuals born in period t is called "generation t." The population of each generation is assumed to be 1, so the population of the economy is constant over time.

There are two types of savings instruments available to individuals: physical capital and bank deposits, and individuals choose one of the two instruments when saving.[6] They face the following trade-off: The rate of return on physical capital (denoted by r^k_{t+1}) is higher than that on bank deposits (denoted by r^d_{t+1}), but the actual rate of return on physical capital that individuals can receive depends on their levels of financial literacy.[7] (A more detailed explanation of this point is provided in the next paragraph.) On the other hand, the rate of return on bank deposits is lower than that on physical capital, but no financial literacy is required to hold them.

Individuals in generation t are endowed with one unit of time in their young period. When saving in the form of physical capital, individual i allocates $0 \leq \phi_{i,t} \leq 1$ of time to acquiring financial literacy and the remaining $1 - \phi_{i,t}$ of time to labor supply. Therefore, the wage income of individual i is $(1 - \phi_{i,t})w_t$, and she allocates part of it to young-age consumption $c^y_{i,t}$ and the rest to savings in the form of physical capital $s_{i,t}$. As mentioned before, the actual return from savings depends on the level of financial literacy. Specifically, we assume that individual i can only receive the fraction $1-\sigma_i/\phi_{i,t}$ of the total return $(1+r^k_{t+1})s_{i,t}$ (i.e., individuals fail to receive $(\sigma_i/\phi_{i,t}) \times (1 + r^k_{t+1})s_{i,t}$.[8]) The unrecovered amount $(\sigma_i/\phi_{i,t}) \times (1 + r^k_{t+1})s_{i,t}$ is smaller when $\phi_{i,t}$ (the time spent on acquiring financial literacy by individual i) is larger or σ_i is smaller. Here, σ_i is an exogenous parameter for individual i's learning ability, and a smaller σ_i corresponds to a higher learning ability. We assume that σ_i is uniformly distributed in the interval $[\underline{\sigma}, \bar{\sigma}]$ and satisfies $0 < \underline{\sigma} \leq \sigma_i \leq \bar{\sigma} < 1$. Accordingly, the individual with the highest (resp. lowest) learning ability has $\underline{\sigma}$ (resp. $\bar{\sigma}$).

[6] We assume that individuals cannot directly hold government bonds. This assumption is not unrealistic. For example, only 1.2% of outstanding government bonds are held by households in Japan.

[7] This assumption is often employed when modeling the individual's investment behavior in financial literacy. See Jappelli and Padula (2013) and Lusardi et al. (2017).

[8] The resources that individuals fail to receive are assumed to be disposed of without being used for either consumption or investment.

We assume that the utility function of individual i in generation t is given by:

$$U_{i,t} = \left(c^y_{i,t}\right)^{\alpha} \left(c^o_{i,t+1}\right)^{1-\alpha},$$

where $c^y_{i,t}$ and $c^o_{i,t+1}$ are the young-age and old-age consumption, respectively. Each individual compares the utility level in the case of choosing physical capital with the utility level in the case of choosing bank deposits, and chooses the one that gives higher utility.[9]

The utility maximization problem in the case of choosing physical capital is

$$\max_{s_{i,t},\phi_{i,t}} U^k_{i,t} \quad \text{s.t.} \quad c^y_{i,t} + s_{i,t} = (1-\phi_{i,t})w_t, \;\; c^o_{i,t+1} = (1-\sigma_i/\phi_{i,t})(1+r^k_{t+1})s_{i,t}.$$

Here, $U^k_{i,t}$, $s_{i,t}$, w_t, and r^k_{t+1} are the utility level when physical capital is chosen, saving in the form of physical capital, wage, and rate of return on physical capital, respectively.

Arranging the first order conditions, we have

$$\left(\frac{\partial U_{i,t}}{\partial s_{i,t}} = 0\right) \qquad s_{i,t} = (1-\alpha)(1-\phi_{i,t})w_t, \tag{2a}$$

$$\left(\frac{\partial U_{i,t}}{\partial \phi_{i,t}} = 0\right) \quad \sigma_i = \frac{(\phi_{i,t})^2}{(1-\alpha)(1-\phi_{i,t})+\phi_{i,t}}, \;\; \frac{\partial \sigma_i}{\partial \phi_{i,t}} > 0, \;\; \frac{\partial^2 \sigma_i}{\partial (\phi_{i,t})^2} > 0, \;\; 0 < \frac{\sigma_i}{\phi_{i,t}} < 1. \tag{2b}$$

From (2b), the function $\phi_{i,t}(\sigma_i)$ satisfies the following properties:

$$\phi_{i,t}(\sigma_i) = \phi(\sigma_i), \qquad \phi(0) = 0, \;\; \phi'(\sigma_i) > 0, \;\; \phi''(\sigma_i) < 0. \tag{3}$$

Figure 2 depicts the function $\phi_{i,t} = \phi(\sigma_i)$. This figure shows that individuals with higher learning ability (i.e., lower σ_i) spend less time acquiring their financial literacy.

Calculating the indirect utility function in this case, we have

$$U^k_{i,t} = \alpha^{\alpha}(1-\alpha)^{1-\alpha}(1+r^k_{t+1})^{1-\alpha}[1-\phi(\sigma_i)]\left(1-\frac{\sigma_i}{\phi(\sigma_i)}\right)^{1-\alpha} w_t. \tag{4}$$

[Figure 2].

[9]Based on essentially the same idea, Spataro and Corsini (2017) formulated the portfolio selection problem of households and investigated the factors that determine the holding of risky assets. However, their model is not a macroeconomic model.

On the other hand, the utility maximization problem in the case of bank deposits is

$$\max_{c^y_{i,t},c^o_{i,t+1}} U^d_{i,t} \quad \text{s.t.} \quad c^y_{i,t} + d_{i,t} = w_t, \;\; c^o_{i,t+1} = (1 + r^d_{t+1})d_{i,t}.$$

Here, $d_{i,t}$ is the savings in the form of bank deposits. The saving function and the indirect utility function in this case are

$$d_{i,t} = (1-\alpha)w_t \tag{5a}$$

$$U^d_{i,t} = \alpha^\alpha(1-\alpha)^{1-\alpha}(1+r^d_{t+1})^{1-\alpha}w_t \tag{5b}$$

Note that $d_{i,t}$ does not depend on an individual's index i. Individual i chooses physical capital if $U^k_{i,t} > U^d_{i,t}$ and bank deposits if $U^k_{i,t} < U^d_{i,t}$. Therefore, the threshold of the learning ability (σ^*_t) at which $U^k_{i,t} = U^d_{i,t}$ holds, can be calculated as follows:

$$1 + r^d_{t+1} = \left(1 + r^k_{t+1}\right)\Phi(\sigma^*_t) \quad \text{where} \;\; \Phi(\sigma^*_t) \equiv [1-\phi(\sigma^*_t)]^{\frac{1}{1-\alpha}}\left(1 - \frac{\sigma^*_t}{\phi(\sigma^*_t)}\right). \tag{6}$$

By calculating $\Phi'(\sigma^*_t)(\equiv \partial\Phi(\sigma^*_t)/\partial\sigma^*_t)$, we can obtain the following (see Appendix A).

$$\Phi'(\sigma^*_t) < 0. \tag{7}$$

If σ_i is lower (resp. higher) than the threshold (σ^*_t), individual i chooses physical capital (resp. bank deposits). Hence, the population (or the share) of individuals who hold physical capital is $((\sigma^*_t - \underline{\sigma})/(\bar{\sigma} - \underline{\sigma}))$, while the population (or the share) of individuals who hold bank deposits is $((\bar{\sigma} - \sigma^*_t)/(\bar{\sigma} - \underline{\sigma}))$.

2.3 Government

The government owes a given amount of debt (denoted by B_t) at the beginning of period t. The government repays debt only by issuing new bonds (i.e., the government plays a "Ponzi game"). We assume that government bonds are the discount bonds at 1 period maturity whose face value is 1. Thus, the government's budget constraint in period t is given by:

$$B_t = q_t B_{t+1}. \tag{8}$$

Here, B_{t+1} is the quantity of government bonds issued in period t, and q_t is their market price. The (gross) government bond yield is equal to $1/q_t$.

2.4 Banks

The banking sector is composed of homogenous banks, the number of which is normalized to 1. From (5a), the total amount of bank deposits in period t is given by:

$$d_t \equiv \int_{\sigma_t^*}^{\bar{\sigma}} d_{i,t} \frac{1}{\bar{\sigma} - \underline{\sigma}} d\sigma_i = (1-\alpha) w_t \frac{\bar{\sigma} - \sigma_t^*}{\bar{\sigma} - \underline{\sigma}}. \tag{9}$$

Each bank invests an exogenous fraction (denoted by $0 < e < 1$) of d_t in physical capital and the rest in government bonds.[10]

The profit of each bank to be realized in period $t+1$ can be expressed as:

$$\pi_{t+1}^d = \left(1 + r_{t+1}^k\right) e d_t + \frac{1}{q_t}(1-e) d_t - \left[\left(1 + r_{t+1}^d\right) + \eta\right] d_t.$$

Here, η is the operating cost per unit of deposit.[11]

The banking sector is assumed to be perfectly competitive ($\pi_{t+1}^d = 0$), so the following holds:

$$1 + r_{t+1}^d = \left(1 + r_{t+1}^k\right) e + \frac{1}{q_t}(1-e) - \eta. \tag{10}$$

This shows that $1 + r_{t+1}^d$ (the interest rate on bank deposits) is the weighted average of $1 + r_{t+1}^k$ (the return rate on physical capital) and $1/q_t$ (the government bond yield) minus η (the bank's operating costs). Thus, in effect, individuals holding bank deposits hold a fraction (e) of their savings in physical capital and the rest in government bonds, paying the cost of η per unit of savings.

2.5 Market equilibrium

In this subsection, we derive the market equilibrium conditions. First, the equilibrium conditions for factor markets in period t are given by $K_t^d = K_t$ and $L_t^d = 1$ (in both equations, the left-

[10] Even when we assume that e depends positively on the difference between the return rate on physical capital $(1 + r_{t+1}^k)$ and the government bond yield $(1/q_t)$, the qualitative results do not change.

[11] We assume the operating cost takes the form of expenditure of the final goods.

hand side (LHS) means demand and the right-hand side (RHS) means supply). Because the population of each generation is equal to one, $\bar{K}_t = K_t$ holds. Substituting these into (1a) and (1b), the equilibrium factor prices are:

$$r_t^k = F_K - \delta, \quad w_t = \theta F_{AL} K_t. \tag{11}$$

Here, both F_K and F_{AL} become constant.[12] Thus, the return rate of physical capital is constant, and the wage is proportional to physical capital stock.

Substituting (11) into (6) and (10), the interest rate on bank deposits $1 + r_{t+1}^d$, and the government bond yield $Q_t (\equiv 1/q_t)$ can be calculated as follows:

$$1 + r_{t+1}^d = (1 + F_K - \delta)\, \Phi(\sigma_t^*) \tag{12a}$$

$$Q_t (\equiv 1/q_t) = \frac{(1 + F_K - \delta)\,[\Phi(\sigma_t^*) - e] + \eta}{1 - e}. \tag{12b}$$

From (12a) and (12b), we can see that when σ_t^* (the threshold value of the individual's learning ability) increases, both $1 + r_{t+1}^d$ and Q_t decline. The reasons for this are as follows. When σ_t^* increases, more individuals choose physical capital as their saving method. In such a situation, $1 + r_{t+1}^d$ must decrease because the return rate of physical capital $(1 + r_{t+1}^k)$ is fixed (see (11)). In addition, $1 + r_{t+1}^d$ is the weighted average of $1 + r_{t+1}^k$ and Q_t from (10), so when r_{t+1}^d falls, Q_t also falls.

Second, the equilibrium condition for the final goods market (i.e., the equilibrium condition between savings and investment) can be expressed as follows:

$$K_{t+1} = \int_{\underline{\sigma}}^{\sigma_t^*} s_{i,t} \frac{1}{\bar{\sigma} - \underline{\sigma}} d\sigma_i + e d_t. \tag{13}$$

Substituting (2a), (9) and (11) into (13), we have

$$G_t \left(\equiv \frac{K_{t+1}}{K_t} \right) = A \left[I(\sigma_t^*, \underline{\sigma}) + e(\bar{\sigma} - \sigma_t^*) \right]. \quad \left(A \equiv \frac{(1 - \alpha)\,\theta F_{AL}}{\bar{\sigma} - \underline{\sigma}} \right). \tag{14}$$

Here, G_t denotes the gross growth rate of physical capital. Since our model is an endogenous

[12] F_K becomes constant, because $F_K\left(K_t^d, \theta \bar{K}_t L_t^d\right) = F_K(K_t, \theta K_t) = F_K(1, \theta)$ holds. Here, the second equality holds because the function F_K is homogenous of degree 0. F_{AL} also becomes constant for the same reason.

growth model of the AK type, it is also the gross economic growth rate. The definition of the function $I\left(\sigma_t^*, \underline{\sigma}\right)$ in (14) is given by

$$I\left(\sigma_t^*, \underline{\sigma}\right) \equiv \int_{\underline{\sigma}}^{\sigma_t^*} [1-\phi(\sigma_i)]d\sigma_i, \quad \left(\frac{\partial I}{\partial \sigma_t^*} = 1-\phi(\sigma_t^*) > 0, \;\; \frac{\partial I}{\partial \underline{\sigma}} = -[1-\phi(\underline{\sigma})] > 0\right). \quad (15)$$

In this paper, we assume the following:

$$\frac{\partial G_t}{\partial \sigma_t^*} > 0 \quad \Leftrightarrow \quad 1-\phi\left(\sigma_t^*\right) - e > 0. \quad (16)$$

(16) implies that when the threshold σ_t^* increases (i.e., when the share of physical capital holders increases), the economic growth rate rises. This assumption seems natural because it supposes that a higher share of direct financing leads to higher economic growth.[13]

Third, the equilibrium condition for the government bond market in period t is given by:

$$q_t B_{t+1} \left(= B_t\right) = (1-e)\, d_t. \quad (17)$$

The LHS of (17) refers to the supply of government bonds, and RHS refers to the banks' demand for them. Substituting (9) and (11) into (17) and arranging, we obtain:

$$b_t \left(\equiv \frac{B_t}{K_t}\right) = (1-e)\, A\left(\bar{\sigma} - \sigma_t^*\right). \qquad \left(A \equiv \frac{(1-\alpha)\,\theta F_{AL}}{\bar{\sigma} - \underline{\sigma}}\right). \quad (18)$$

Here, b_t is the ratio of government debt to physical capital at the beginning of period t, which is historically given. The threshold σ_t^* in period t is determined to satisfy (18).[14]

Finally, by substituting (14) and (18) into (8) and rearranging, we derive the following autonomous difference equation for σ_t^*.

$$\sigma_{t+1}^* = \bar{\sigma} - \frac{Q\left(\sigma_t^*\right)\left(\overline{\sigma} - \sigma_t^*\right)}{G\left(\sigma_t^*\right)}. \quad (19)$$

Here, the definitions of $Q\left(\sigma_t^*\right)$ and $G\left(\sigma_t^*\right)$ are given by (12b) and (14), respectively.

Because the value of σ_t^* in the initial period t is determined by (18), the subsequent path

[13]Without this assumption, the analysis of the existence and stability of balanced growth paths becomes rather complicated.

[14]Namely, σ_t^* is not a jump variable at the initial period t.

is also uniquely determined by (19). Once the path of σ_t^* is determined, $1 + r_{t+1}^d$ (the interest rate on bank deposits), Q_t (the government bond yield), and G_t (the economic growth rate) are determined from (12a), (12b), and (14).

3 Existence and stability of BGPs

In this section, we investigate the existence and stability of balanced growth paths (BGPs) both theoretically and numerically. Here, a BGP is a growth path through which σ_t^* (the threshold of the individual's learning ability) is constant over time.

First, we discuss the existence of the BGPs. From (19), it is easy to see that $\sigma_t^* = \bar{\sigma}$ is one of the BGPs. Since all individuals choose physical capital as a means of saving in this BGP, the ratio of government debt to physical capital in this BGP is zero.

In the other BGP, where $\sigma_t^* \neq \bar{\sigma}$ holds, (19) becomes

$$G\left(\sigma^*\right) = Q\left(\sigma^*\right), \tag{20}$$

where the LHS of (20) is the (gross) economic growth rate and the RHS is the (gross) government bond yield. From (7), (12b), and (16), we have

$$\frac{\partial G}{\partial \sigma^*} = A[1 - \phi(\sigma^*) - e] > 0, \;\; \frac{\partial Q}{\partial \sigma^*} = \frac{(1 + F_K - \delta)\Phi'(\sigma^*)}{1 - e} < 0. \tag{21}$$

Here, $\partial G/\partial\sigma^* > 0$ holds by assumption (16). In contrast, the reason why $\partial Q/\partial\sigma^* < 0$ holds is that a rise in σ^* lowers the deposit interest rate (see (12a)) and the government bond yield is linked to the deposit interest rate (see (10)).

In the following we assume

$$G\left(\underline{\sigma}\right) < Q\left(\underline{\sigma}\right), \quad G\left(\bar{\sigma}\right) > Q\left(\bar{\sigma}\right). \tag{22}$$

Under (21) and (22), (20) can be depicted as in Figure 3. It is clear from this figure that the BGP exists uniquely in the range $\underline{\sigma} < \sigma^* < \bar{\sigma}$. In this BGP, the ratio of government debt to physical capital is positive because there are individuals who hold bank deposits. Hence, in our model, there are two BGPs: one where the ratio of government debt to physical capital is zero and the other where it is positive.

[Figure 3].

Next, we examine the stability of the BGPs. We define the RHS of (19) as $H(\sigma_t^*)$. By (21) and (22), we can easily show that the function $H(\sigma_t^*)$ has the following properties:

$$H(\underline{\sigma}) < \underline{\sigma}, \quad H(\bar{\sigma}) = \bar{\sigma}, \quad H'(\sigma_t^*) > 0, \quad H'(\bar{\sigma}) < 1. \tag{23}$$

From (23) and the uniqueness of the BGP that satisfies $\underline{\sigma} < \sigma_t^* < \bar{\sigma}$, the phase diagram of (19) can be drawn as the upper half of Figure 4, from which we can see that E^s (the BGP with $\sigma_t^* = \bar{\sigma}$) is stable, whereas E^u (the BGP with $\underline{\sigma} < \sigma_t^* < \bar{\sigma}$) is unstable. To avoid confusion, we denote the value of σ_t^* in the stable BGP as $\sigma_s^* (= \bar{\sigma})$ and in the unstable BGP as σ_u^*.

The lower half of Figure 4 depicts the relationship between σ_t^* and b_t (the ratio of government debt to physical capital), as shown by (18). The ratio of government debt to physical capital in the unstable BGP is given by

$$b_u^* = (1-e)A(\bar{\sigma} - \sigma_u^*). \quad \left(A \equiv \frac{(1-\alpha)\,\theta F_{AL}}{\bar{\sigma} - \underline{\sigma}} \right) \tag{24}$$

This value corresponds to the maximum sustainable ratio of government debt to physical capital. If the initial value of b_t (which is historically given) is smaller than b_u^*, the initial value of σ_t^* becomes larger than σ_u^*, and σ_t^* converges to its upper limit $\bar{\sigma}$. In this case, the economy converges to the stable BGP, along which the value of b_t will decline over time. Conversely, if the initial value of b_t is larger than b_u^*, the initial value of σ_t^* becomes smaller than σ_u^*, and σ_t^* will decline over time. In this case, the government will go bankrupt within a finite period.

[Figure 4]

Summarizing the results so far, we have the following proposition.

Proposition 1. *Under assumptions* (16) *and* (22)*, the dynamics of* σ_t^* *(the threshold of the individual's learning ability) shown in* (19) *has one stable BGP, where the ratio of government debt to physical capital is 0 and one unstable BGP where its ratio is positive. The economic growth rate and the ratio of government debt to physical capital in both BGPs are given as follows:*

$$(\text{unstable BGP}) \quad G_u^* = A\left[I\left(\sigma_u^*, \underline{\sigma}\right) + e\left(\bar{\sigma} - \sigma_u^*\right)\right], \quad b_u^* = (1-e)\,A\left(\bar{\sigma} - \sigma_u^*\right) \tag{25a}$$

$$(\text{stable BGP}) \quad G_s^* = AI\left(\bar{\sigma}, \underline{\sigma}\right), \quad b_s^* = 0. \tag{25b}$$

Before proceeding to further analysis, we confirm the validity of the above result with a numerical example. We set the benchmark numerical values of the parameters as shown in Table 2. The two new parameters $(\gamma,\ \Gamma)$ in the table occur from the specification of the production function as $Y = \Gamma K^{\gamma}\left(KL\right)^{1-\gamma}$.[15]

[Table 2]

Details regarding the numerical settings of the parameters are as follows. For the parameter α in the utility function, we set $\alpha = 0.69$ because the discount factor should satisfy $(1-\alpha)/\alpha = 0.973^{30} \approx 0.44$ (see Song et al., 2012). The parameter γ in the production function is set to $\gamma = 0.37$ by averaging the values of the US ($\gamma = 0.35$), the EU ($\gamma = 0.38$), and Japan ($\gamma = 0.38$).[16] For the capital depreciation rate δ, we assume $\delta = 1$ (i.e., the case of full depreciation).[17] Under this parameter setting for the production function, we have $1 + r^k = \Gamma\gamma = \Gamma \times 0.37$. Assuming that the average annual rate of return on capital is 4% (see Trabandt and Uhlig (2011)), we have $1 + r^k = 1.04^{30} \approx 3.24$, which means $\Gamma \approx 8.76$. Therefore, the marginal productivity of labor can be calculated as $F_{AL} = \Gamma\left(1-\gamma\right) = 5.52$. For the parameter e, we set $1 - e = 0.2$, because the ratio of public debt to total deposits in the private banks in the US, the EU, and Japan is 0.140, 0.212, and 0.192, respectively, and their average value is approximately 0.2.[18] The parameter η (i.e., the operating cost of private banks) is set to $\eta = 1.69$ to satisfy (10).[19] Finally, for the

[15] In this specification of the production function, $\theta = 1$ is assumed.

[16] See Trabandt and Uhlig (2011) for the values of the US and the EU, and Hansen and İmrohoroğlu (2016) for the value of Japan

[17] It is natural assumption because we regard one period as around 30 years.

[18] See International Financial Statistics (IFS). Here, the private banking sector refers to "Other Depository Corporations" (i.e., depository corporations other than the central bank) in the statistics. We also calculate the outstanding government debt held by the private banking sector as the sum of "Claims on Central Government" and "Claims on State and Local Governments" in the statistics, and outstanding deposits as the sum of "Transferable Deposits Included in Broad Money" and "Other Deposits Included in Broad Money" (accessed on September 17, 2021).

[19] The average annual deposit interest rate in Japan, the EU, Canada, and the US is about 0.9%, so we have $1 + r^d = 1.009^{30} \approx 1.31$. The average annual yield on government bonds in Japan, the EU, Canada, and the US is about 2.4%, so we have $1/q = 1.024^{30} \approx 2.04$. We have referred to the International Financial Statistics (IFS) for these data (accessed on September 17, 2021). On the other hand, for the return rate on capital, we use the value $1 + r^k \approx 3.24$ derived earlier. Therefore, the value of η that satisfies (10) is $\eta = 1.69$.

parameters $(\bar{\sigma}, \underline{\sigma})$, we set $(0.07, 0.04)$ such that the two assumptions (16) and (22) are satisfied, and $Q(\sigma_t^*)$ in (12b) becomes positive.

We confirm that the analytical results can be reproduced under the above parameter settings. Figure 5-(a) represents the value of $\partial G_t / \partial \sigma_t^* = 1 - \phi(\sigma_t^*) - e$, and it checks whether assumption (16) holds. This figure shows that assumption (16) holds for all possible ranges of σ_i. Figure 5-(c) illustrates the existence of an unstable BGP: $\sigma_u^* \in (\underline{\sigma}, \bar{\sigma})$ ((20)), which is in line with Figure 3. Figures 5-(b) and 5-(d) illustrate respectively the stability of the two BGPs ((19)) and the relationship between σ_t^* and b_t ((18)), and are in line with the upper half and lower half of Figure 4.

[Figures 5].

From Figure 5-(c), we can confirm that the value of σ_u^* in the unstable BGP is 0.0572 under the benchmark numerical settings. Accordingly, from Figure 5-(d), the maximum sustainable ratio of public debt to physical capital is $b_u^* = 0.1464$. On the other hand, the actual average ratio of public debt to physical capital for the EU, the US, and Japan is approximately 0.11.[20] If we use $b_0 = 0.11$ (the initial value of b_t), the economy converges to the stable BGP because $b_0 (= 0.11) < b_u^* (= 0.1464)$.

[Figure 6].

Figures 6-(a), 6-(b), and 6-(c) show the evolution of σ_t^*, G_t and Q_t during the transition process when $b_0 = 0.11$ with the benchmark parameters in Table 2. From these figures, we can see that σ_t^* converges to $\bar{\sigma}$ within 10 periods, and the economic growth rate increases and the government bond yield decreases during the transition. The changes in these variables are also consistent with the analytical results (see (21)).

[20]According to AMECO database, the average value of Y/K (i.e., the ratio of GDP to physical capital) for Japan, the EU, the US, and Canada is about 0.4. On the other hand, the average value of B/Y (i.e., the ratio of public debt to GDP) for those four countries is approximately 1.41. Here, we used the OECD database for the values for Japan, the US, and the EU, and the AMECO database for the value for Canada. (Accessed on September 17, 2021). Therefore, the average value of B/K (i.e., the ratio of public debt to physical capital) is $0.4 \times 1.41 = 0.564$. According to Arslanalp and Tsuda (2014), the share of government bonds held by domestic private banks is approximately 20%, so the ratio of public debt held by the private banking sector to physical capital is $0.564 \times 0.2 \approx 0.11$.

4 Effect of changes in the learning ability of individuals

In this section, we examine, both analytically and numerically, the effect of a decrease in σ_i (i.e., an increase in the individual's learning ability) on economic growth and fiscal sustainability. Here, a situation in which σ_i decreases uniformly can be interpreted as one in which the average financial literacy of individuals increases. As mentioned briefly in the introduction, with the increasing importance of financial literacy due to the increasing complexity of financial markets and rising life expectancy, a growing number of countries have introduced educational programs that improve financial literacy, especially for younger generations (OECD, 2014). This section analyzes the implications of such a policy on economic growth and fiscal sustainability.

We consider a situation in which σ_i changes uniformly by $d\sigma(>0)$ (i.e., both the lower and upper bounds of the uniform distribution of σ_i change by $d\underline{\sigma} = d\bar{\sigma} = d\sigma$). From (20), the effect of such a change on σ_u^* (the threshold of the individual's learning ability in the unstable BGP) can be calculated as follows:

$$\frac{d\sigma_u^*}{d\sigma} = -\frac{\frac{\partial G_u^*}{\partial \sigma}}{\frac{\partial G_u^*}{\partial \sigma_u^*} - \frac{\partial Q_u^*}{\partial \sigma_u^*}} = \frac{A\left[1-\phi\left(\underline{\sigma}\right)-e\right]}{A\left[1-\phi\left(\sigma_u^*\right)-e\right]-Q'\left(\sigma_u^*\right)} > 0, \tag{26a}$$

$$\text{where} \quad \frac{\partial G_u^*}{\partial \sigma} = -A\left[1-\phi\left(\underline{\sigma}\right)-e\right] < 0, \tag{26b}$$

$$\frac{\partial G_u^*}{\partial \sigma_u^*} = A\left[1-\phi\left(\sigma_u^*\right)-e\right] > 0, \quad \frac{\partial Q_u^*}{\partial \sigma_u^*} = \frac{(1+F_K-\delta)\Phi'(\sigma_u^*)}{1-e} < 0. \tag{26c}$$

(26a) shows that a uniform decrease in σ_i lowers σ_u^*. Note that because both $(\overline{\sigma}, \underline{\sigma})$ and σ_u^* move in the same direction, it is ambiguous whether the share of physical capital holders ($= (\sigma_u^* - \underline{\sigma})/(\bar{\sigma} - \underline{\sigma})$) in the new unstable BGP increases or decreases.

Next, we examine the effect of the change in σ_i on G_u^* (the economic growth rate in the unstable BGP) and G_s^* (the economic growth rate in the stable BGP). From (25a) and (25b), we have

$$\frac{dG_u^*}{d\sigma} = \overbrace{\frac{\partial G_u^*}{\partial \sigma_u^*}\frac{\partial \sigma_u^*}{\partial \sigma}}^{\text{indirect effect}(+)} + \overbrace{\frac{\partial G_u^*}{\partial \sigma}}^{\text{direct effect}(-)} = \frac{A\left[1-\phi\left(\underline{\sigma}\right)-e\right]Q'\left(\sigma_u^*\right)}{A\left[1-\phi\left(\sigma_u^*\right)-e\right]-Q'\left(\sigma_u^*\right)} < 0, \tag{27a}$$

$$\frac{dG_s^*}{d\sigma} = -A\left[\phi\left(\overline{\sigma}\right)-\phi\left(\underline{\sigma}\right)\right] < 0. \tag{27b}$$

(27a) and (27b) show that a uniform decrease in σ_i raises both G_u^* and G_s^*. The reason for this result is as follows: (27a) indicates that in the unstable BGP a uniform decrease in σ_i affects G_u^* both indirectly (i.e., via the impact on σ_u^*) and directly (i.e., not via the impact on σ_u^*). From (26a), (26b), and (26c), the sign of the indirect effect is positive, while that of the direct effect is negative. Because the negative direct effect exceeds the positive indirect effect in our model, the sign of $dG_u^*/d\sigma$ becomes negative. (27b), on the other hand, indicate that a uniform decrease in σ_i in the stable BGP reduces the learning time $\phi(\sigma_i)$ and raises both savings and G_s^*.

Finally, we examine the effect of the change in σ_i on b_u^* (the ratio of public debt to physical capital in the unstable BGP, or the maximum sustainable ratio of public debt to physical capital). From (25a) and (26a), we can derive the following relationship (see Appendix B for details of the derivation):

$$\frac{db_u^*}{d\sigma} \gtreqless 0 \quad \Leftrightarrow \quad -\frac{dG_{u,t}^*}{d\sigma} \lesseqgtr -\frac{dQ_{u,t}^*}{d\sigma}. \tag{28}$$

Here, $dG_{u,t}^*/d\sigma$ and $dQ_{u,t}^*/d\sigma$ means that when σ_i changes in period t in the unstable BGP, how much the economic growth rate and government bond yield will change in the same period t (not in the new unstable BGP). As shown in Appendix B, if σ_i declines in period t, both the economic growth rate and the government bond yield in the same period t increase (i.e., $dG_{u,t}^*/d\sigma < 0$ and $dQ_{u,t}^*/d\sigma < 0$). (28) indicates that the sign of $db_u^*/d\sigma$ is ambiguous in general, and it depends on whether $-dG_{u,t}^*/d\sigma$ or $-dQ_{u,t}^*/d\sigma$ is larger.

Consider the case where $-dG_{u,t}^*/d\sigma < -dQ_{u,t}^*/d\sigma$ holds, because we will show numerically below that this case indeed is realized. Then, (28) shows that a decline in σ_i reduces b_u^*. The reasons for this are as follows. When σ_i declines in period t in the unstable BGP, the following two cases can occur: In the first case, the economy may converge to a stable BGP. In the second case, it may proceed to a divergent path and eventually go bankrupt. The latter case is more likely to occur because $-dG_{u,t}^*/d\sigma < -dQ_{u,t}^*/d\sigma$ shows that an increase in the government bond yield becomes larger than that in the economic growth rate, inducing the ratio of government debt to physical capital to explode. Thus, a decline in σ_i reduces the maximum sustainable ratio of government debt to physical capital b_u^*.

[Figure 7].

We check the sign of $db_u^*/d\sigma$ under plausible parameter values. Figure 7 shows that a uniform decrease in σ_i lowers b_u^* (i.e., $db_u^*/d\sigma > 0$ holds) under the benchmark numerical settings of the parameters in Table 2.[21] This numerical result indicates that as individuals become more financially literate, the maximum sustainable ratio of public debt to physical capital declines (i.e., fiscal sustainability worsens). Therefore, if the actual ratio of public debt to physical capital in the initial period is close to its maximum sustainable value, a uniform decrease in σ_i will cause σ_t^* to decline (and accordingly, cause b_t to rise) over time, and the government will go bankrupt within a finite period of time. On the other hand, if the actual ratio of public debt to physical capital in the initial period is sufficiently smaller than its maximum sustainable value, the economy still converges to the stable BGP even when σ_i decreases.

[Figure 8].

Figures 8-(a), 8-(b), and 8-(c) show the dynamic paths of the economy when the ratio of public debt to physical capital at the initial period (i.e., period $t = 0$) is given by 0.11 and the values of $(\bar{\sigma}, \underline{\sigma})$ are respectively (i) $(0.07, 0.04)$, (ii) $(0.08, 0.05)$, and (iii) $(0.06, 0.03)$. Case (i) is the benchmark case, and the dynamic path for this case has already been explained in Section 3. Case (ii) is the case where the individual's learning ability is low compared to the benchmark case, and case (iii) is the case where the individual's learning ability is high compared to the benchmark case. When the individual's learning ability is relatively high (case (iii)), σ_t^* declines sharply (accordingly, b_t rises sharply) and reaches its lower bound ($\underline{\sigma}$) at period $t = 3$. Along this path to bankruptcy, the government bond yield grows rapidly and the economic growth rate declines deeply. By contrast, when the individual's learning ability is relatively low (case (ii)), σ_t^* rises (accordingly, b_t declines), and σ_t^* reaches the stable BGP at period $t = 3$. Along this stable path, the government bond yield declines and the economic growth rate rises steadily.

Figure 9 illustrates the welfare level of each generation in the benchmark case (i.e., $(\bar{\sigma}, \underline{\sigma}) = (0.07, 0.04)$) and the case when the individual's learning ability is relatively low (i.e., $(\bar{\sigma}, \underline{\sigma}) = (0.08, 0.05)$).[22] Here, we define the welfare of generation t as

$$W_t = \frac{1}{\bar{\sigma} - \underline{\sigma}} \left[\int_{\underline{\sigma}}^{\sigma_t^*} U_{i,t}^k d\sigma_i + \int_{\sigma_t^*}^{\bar{\sigma}} U_{i,t}^d d\sigma_i. \right]$$

[21]This result is robust even when the value of e (i.e., the ratio of physical capital investment to total deposits in the banking sector) is 0.75 or 0.85.

[22]In the case where the learning ability is relatively high (i.e., $(\bar{\sigma}, \underline{\sigma}) = (0.06, 0.03)$), the government goes bankrupt within a finite period and the behavior of the economy after that is not modeled. The welfare analysis in this case is thus omitted.

Figure 9 shows that when the learning ability declines compared to the benchmark case, the welfare of each generation decreases, and that the size of decline in welfare becomes larger over time. This is mainly because a decline in the learning ability lowers the economic growth rate through more limited participation in the capital market, which lowers wage levels of future generations.

[Figure 9]

Summarizing the results in this section, we have the following proposition.

Proposition 2. *Suppose that σ_i decreases uniformly (i.e., the learning ability or financial literacy of individuals increases). Then, the following holds:.*

(1) Such a change raises the economic growth rates of both unstable and stable BGPs.

(2) The effect of such a change on b_u^* (the maximum sustainable ratio of public debt to physical capital) is ambiguous in general, but under the parameter settings in Table 2 such a change lowers b_u^*.

(3) Under the parameter settings in Table 2, such a change improves the welfare of each generation and the degree of welfare improvement increases as time passes, if the economy converges to the stable BGP by such a change.

This proposition shows that an improvement in the average level of an individual's financial literacy certainly increases the potential economic growth, whereas it worsens fiscal sustainability and increases the likelihood of the government's bankruptcy. This result implies that in a country with a high ratio of public debt to GDP, such as Japan, public policy to improve individuals' financial literacy should be combined with fiscal consolidation to avoid the government's bankruptcy.

5 Effect of changes in banks' asset investment behavior

In this section, we examine, both analytically and numerically, how the change in banks' asset investment behavior (specifically, the change in the parameter e, i.e., the ratio of physical capital investment to total deposits) affects economic growth and fiscal sustainability. It seems likely that

(i) an increase in e will improve economic growth because it increases physical capital investment by banks, while (ii) an increase in e worsens fiscal sustainability because it reduces the amount of government debt underwritten by banks. The purpose of this section is to examine whether such a prediction is correct.

First, we examine the effect of an increase in e on σ_u^* (the threshold of an individual's learning ability in the unstable BGP).

By totally differentiating (20), we have

$$\frac{d\sigma_u^*}{de} = -\frac{\frac{\partial G_u^*}{\partial e} - \frac{\partial Q_u^*}{\partial e}}{\frac{\partial G_u^*}{\partial \sigma_u^*} - \frac{\partial Q_u^*}{\partial \sigma_u^*}}, \tag{29a}$$

$$\text{where} \quad \frac{\partial G_u^*}{\partial e} = A\left(\overline{\sigma} - \sigma_u^*\right) > 0, \quad \frac{\partial Q_u^*}{\partial e} = \frac{-\left(1 + F_K - \delta\right)\left[1 - \Phi\left(\sigma_u^*\right)\right] + \eta}{\left(1-e\right)^2}, \tag{29b}$$

$$\frac{\partial G_u^*}{\partial \sigma_u^*} = A\left[1 - \phi\left(\sigma_u^*\right) - e\right] > 0, \quad \frac{\partial Q_u^*}{\partial \sigma_u^*} = \frac{\left(1 + F_K - \delta\right)\Phi'\left(\sigma_u^*\right)}{1-e} < 0. \tag{29c}$$

Because the sign of $\partial Q_u^*/\partial e$ in (29b) is ambiguous, so is the sign of $d\sigma_u^*/de$ in general. However, the sign of $d\sigma_u^*/de$ is found to be negative if we assume $\partial Q_u^*/\partial e < 0$ as follows:

$$\frac{d\sigma_u^*}{de} < 0 \quad \text{if} \quad \frac{\partial Q_u^*}{\partial e} < 0 \quad \Leftrightarrow \quad \eta < (1 + F_K - \delta)[1 - \Phi(\sigma_u^*)] \tag{30}$$

Figure 10 shows that the assumption (i.e., $\eta < (1 + F_K - \delta)[1 - \Phi(\sigma_u^*)]$) in (30) is satisfied under the benchmark parameter settings in Table 2.[23] Thus, in the following analyses, we will assume (30) (i.e., an increase in e reduces the share of individuals who choose physical capital as a means of saving in the unstable BGP).

[Figure 10].

Next, we examine the effect of an increase in e on G_u^* (the economic growth rate of the unstable BGP). Before addressing this, note that G_s^* (the economic growth rate of the stable BGP) is independent of e (see (25b)) because in the stable BGP, all individuals hold their savings in the form of physical capital (i.e., there are no bank deposits in the economy). Thus, we focus

[23]More precisely, Figure 10 shows that the assumption in (30) holds not only for $(\bar{\sigma}, \underline{\sigma}) = (0.07, 0.04)$ (i.e., the benchmark case), but also for $(\bar{\sigma}, \underline{\sigma}) = (0.08, 0.05)$ and $(\bar{\sigma}, \underline{\sigma}) = (0.06, 0.03)$. We can also confirm through numerical calculations that this assumption holds even in the case of $e = 0.75$ or 0.85.

on G_u^*. From (25a), the effect of an increase in e on G_u^* can be calculated as follows:

$$\frac{dG_u^*}{de} = \overbrace{\frac{\partial G_u^*}{\partial \sigma_u^*}\frac{\partial \sigma_u^*}{\partial e}}^{\text{indirect effect}(-)} + \overbrace{\frac{\partial G_u^*}{\partial e}}^{\text{direct effect}(+)} = A\left[1-\phi\left(\sigma_u^*\right)-e\right]\frac{\partial \sigma_u^*}{\partial e} + A\left(\bar{\sigma}-\sigma_u^*\right). \tag{31}$$

From (31), we can see that an increase in e has the following two effects: the indirect effect on G_u^* via the impact on σ_u^* (i.e., the share of physical capital holder) and the direct effect on G_u^*, not via the impact on σ_u^*. The sign of the direct effect on G_u^* is positive (by (29b)), while that of the indirect effect on G_u^* is negative (by (29c) and (30)). This is because an increase in e raises the physical capital investment by banks directly, whereas it reduces the share of individuals who choose physical capital as a means of saving. Therefore, the total effect is generally ambiguous. In the following, we examine the sign of dG_u^*/de using numerical calculations.

Figure 11-(b) shows how G_u^* reacts to the changes in e under the benchmark parameter settings in Table 2. We can find that an increase in e raises G_u^* in the range of e from 0.75 to 0.85. This result implies that the positive direct effect exceeds the negative indirect effect in (31). This result still holds when the value of $(\bar{\sigma}, \underline{\sigma})$ takes from $(0.07, 0.04)$ to $(0.08, 0.05)$. However, if the value of $(\bar{\sigma}, \underline{\sigma})$ is set to $(0.06, 0.03)$, the above result changes. Figure 11-(a) illustrates this case and shows that there is a value of e that minimizes G_u^*. If e is larger (resp. smaller) than that value, an increase in e raises (resp. lowers) G_u^*. Therefore, the effect of the change in e on G_u^* varies depending on the value of $(\bar{\sigma}, \underline{\sigma})$.

[Figure 11]

Finally, we examine the effect of the change in e on b_u^* (i.e., the maximum sustainable ratio of public debt to physical capital). From (25a), we can derive the following:

$$\frac{db_u^*}{de} = \overbrace{\frac{\partial b_u^*}{\partial \sigma_u^*}\frac{\partial \sigma_u^*}{\partial e}}^{\text{indirect effect}(+)} + \overbrace{\frac{\partial b_u^*}{\partial e}}^{\text{direct effect}(-)} = -A(1-e) \times \overbrace{\frac{\partial \sigma_u^*}{\partial e}}^{(-)\text{ by (30)}} - A(\bar{\sigma}-\sigma_u^*). \tag{32}$$

As before, the effect of an increase in e on b_u^* can be divided into indirect and direct effects. From (30), the indirect effect on b_u^* is positive, while the direct effect on b_u^* is negative, so the total effect is generally ambiguous.[24]

Figure 12-(b) illustrates how b_u^* reacts to the changes in e under the benchmark parameter settings in Table 2. From this figure, we find that a value of e that maximizes b_u^* exists in the range of e from 0.75 to 0.85.[25][26] This result is consistent with aforementioned result that a value of e that minimizes G_u^* exists, because a larger outstanding public debt induces a smaller economic growth rate due to its crowding-out effect on physical capital. Figures 12-(c) and 12-(a) show the graphs when the values of $(\bar{\sigma}, \underline{\sigma})$ are re-set from the benchmark case of $(0.07, 0.04)$ to $(0.08, 0.05)$ and $(0.06, 0.03)$, respectively. When $(\overline{\sigma}, \underline{\sigma}) = (0.08, 0.05)$ (i.e., when the average level of learning ability (or financial literacy) is relatively low), an increase in e lowers b_u^* in the range of e from 0.75 to 0.85. This is because the direct effect exceeds the indirect effect (see (32)) in this case. On the other hand, when $(\overline{\sigma}, \underline{\sigma}) = (0.06, 0.03)$ (i.e., when the average level of learning ability (or financial literacy) is relatively high), an increase in e raises b_u^* in the range of e from 0.75 to 0.85. This is because the indirect effect exceeds the direct effect (see (32)) in this case.[27]

[Figure 12].

Summarizing the results of this section, we have the following proposition.

Proposition 3. *Suppose that banks raise* e *(the ratio of physical capital investment to total deposits). The following then holds:*

(1) Such a change reduces the share of physical capital holders in the unstable BGP under (30). (30) *is satisfied under the parameter settings in Table 2.*

(2) The effect of such a change on G_u^* *(the economic growth rate in the unstable BGP) is ambiguous in general, but under the parameter settings in Table 2, such a change raises*

[24]For the effect of an increase in e on b_u^*, we can derive a similar condition as (28):

$$\frac{db_u^*}{de} \gtreqless 0 \Leftrightarrow \frac{dG_{u,t}^*}{de} \gtreqless \frac{dQ_{u,t}^*}{de}.$$

Here, LHS (resp. RHS) of the second inequality means that when e rises in period t in the unstable BGP, how much the economic growth rate (resp. government bond yield) changes in the same period t. This relationship shows that if the change in the economic growth rate is larger (resp. smaller) than the change in government bond yield, an increase in e increases (resp. lowers) b_u^*.

[25]From (31) and (32), we can see that the value of e that minimizes G_u^* (i.e., the value of e that satisfies $dG_u^*/de = 0$) is not equal to the value of e that maximizes b_u^* (i.e., the value of e that satisfies $db_u^*/de = 0$).

[26]This result will still hold when the value of $(\bar{\sigma}, \underline{\sigma})$ is varied slightly (e.g. $(\bar{\sigma}, \underline{\sigma}) = (0.065, 0.035)$ or $(0.075, 0.045)$).

[27]These results do not necessarily mean that a value of e that maximizes b_u^* does not exist when $(\bar{\sigma}, \underline{\sigma})$ is $(0.08, 0.05)$ or $(0.06, 0.03)$. Even under these values of $(\bar{\sigma}, \underline{\sigma})$, a value of e that maximizes b_u^* will exist if the range of e is set to be wider.

G_u^*. Furthermore, when the value of $(\bar{\sigma}, \underline{\sigma})$ (upper and lower bounds of σ_i) is low compared to the benchmark case (i.e., when the individual's learning ability is relatively high), there exists a value of e that minimizes G_u^*.

(3) The effect of such a change on b_u^* (the maximum sustainable ratio of public debt to physical capital) is ambiguous in general. The following results hold based on numerical calculations.

(a) When the value of $(\bar{\sigma}, \underline{\sigma})$ is the benchmark value shown by Table 2, there exists a value of e that maximizes b_u^*. (b) When the value of $(\bar{\sigma}, \underline{\sigma})$ is high compared to the benchmark case (i.e., the individual's learning ability is relatively low), such a change lowers b_u^*. (c) When the value of $(\bar{\sigma}, \underline{\sigma})$ is low compared to the benchmark case (i.e., the individual's learning ability is relatively high), such a change raises b_u^*.

As mentioned at the beginning of this section, it seems likely that an increase in e promotes economic growth, while it worsens fiscal sustainability. This is because an increase in e raises the physical capital investment by banks, while it reduces the amount of government bonds underwritten by banks (i.e., the direct effect). However, Proposition 3 shows that the result can be more complicated than such a prediction. This is because an increase in e also has the indirect effect of reducing the share of physical capital holders in the new unstable BGP, which has the opposite impact to the direct effect. Depending on which effect dominates, the effects of the change in e on economic growth and fiscal sustainability vary.

The following result is particularly noteworthy: When the average financial literacy of individuals is relatively low, an increase in e raises b_u^* (i.e., when banks decrease the amount of government bonds they underwrite, fiscal sustainability conversely improves). This result implies that in countries where the average financial literacy of individuals is relatively low, the need for fiscal consolidation will conversely decrease (resp. increase) if banks decrease (resp. increase) the number of government bonds they underwrite.

6 Conclusion

We examined the implications of limited stock market participation (the stylized fact that only a fraction of households directly own stocks) for economic growth and, in particular, fiscal sustainability (or the maximum sustainable level of government debt). Constructing an OG model where (1) individuals can choose between two types of savings (i.e., physical capital with high returns but costly to hold versus bank deposits with low returns but no costs), and (2) banks invest part of the total deposits in physical capital and use the rest to underwrite government debt, we mainly showed the following: First, an increase in the average level of an individual's financial literacy worsens fiscal sustainability while promoting economic growth under the benchmark numerical settings of parameters. With the increasing importance of financial literacy due to the increasing complexity of financial markets and rising life expectancy, a growing number of countries have introduced educational programs that improve financial literacy. The above result implies that while such a policy certainly has the positive effect of raising the potential economic growth rate, it also has the negative effect of worsening fiscal sustainability and increasing the likelihood of the government's bankruptcy. Second, the effect of the change in banks' asset investment behavior on fiscal sustainability varies depending on the average level of financial literacy. Specifically, if banks decrease (resp. increase) the number of government bonds they underwrite, fiscal sustainability conversely improves (resp. worsens) in the case where the average financial literacy of individuals is relatively low. This result implies that in countries where the average financial literacy of individuals is relatively low, the need for fiscal consolidation will conversely decrease (resp. increase) if banks decrease (resp. increase) the amount of government bonds they underwrite.

In this study, we adopted an ad hoc formulation of the banks' asset investment behavior to simplify the analysis, but it would be more desirable to make the optimization behavior of banks explicit.[28] This is an issue for future research.

Acknowledgments

The authors thank an anonymous referee for their valuable comments. The first author was supported by grants-in-aid from the Ministry of Education, Culture, Sports, Science, and Technology of the Japanese government KAKENHI (grant number 20K13455).

[28]Ono (2019) is an example of such an analysis.

References

[1] Allen F, Gale D (1994) Limited market participation and volatility of asset prices. Am Econ Rev 84(4):933-955

[2] Alvarez F, Lucas R, Weber W (2001) Interest rates and inflation. Am Econ Rev 91(2):219-225

[3] Arai R (2011) Productive government expenditure and fiscal sustainability. FinanzArchiv 67(4):327-351

[4] Arslanalp S, Tsuda T (2014) Tracking global demand for advanced economy sovereign debt. IMF Econ Rev 62(3):430-464

[5] Barro R (1990) Government spending in a simple model of endogenous growth. J Polit Econ 98(5):103-125

[6] Bilbiie F (2008) Limited asset market participation, monetary policy and (inverted) aggregate demand logic. J Econ Theory 140(1):162-196

[7] Bräuninger M (2005) The budget deficit, public debt, and endogenous growth. J Public Econ Theory 7(5):827-840

[8] Brown R, Ivkovic Z, Smith A, Weisbenner P (2008) Neighbours matter: Causal community effects and stock market participation. J Finance 63(3):1509-1531

[9] Campbell J (2006) Household finance. J Finance 61(4):1553-1604

[10] Chalk N (2000) The sustainability of bond-finance deficits: An overlapping generations approach. J Monet Econ 45(2):293-328

[11] Christelis D, Jappelli T, Padula M (2010) Cognitive abilities and portfolio choice. Eur Econ Rev 54(1):18-38

[12] Christiano L, Eichenbaum M (1995) Liquidity effects, monetary policy, and the business cycle. J Money Credit Bank 27(4):1113-1136

[13] Constantinides M, Donaldson B, Mehra R (2002) Junior can't borrow: A new perspective on the equity premium puzzle. Q J Econ 117(1):269-296

[14] Diamond P (1965) National debt in a neoclassical growth model. Am Econ Rev 55:1126-1150

[15] Farmer K, Zotti J (2010) Sustainable government debt in a two-good, two-country overlapping generations model. Int Rev Econ 57(3):289-316

[16] Fujiki H, Hirakata N, Shioji E (2012) Aging and household stockholdings: Evidence from Japanese household survey data. Discussion Paper No. 2012-E-17, Institute for Monetary and Economic Studies

[17] Futagami K, Morita Y, Shibata A (1993) Dynamic analysis of an endogenous growth model with public capital. Scand J Econ 95(4):607-625

[18] Galí J, López-Salido D, Vallés J (2004) Rule of thumb consumers and the design of interest rate rules. J Money Credit Bank 36(4):739-764

[19] Grossman G, Yanagawa N (1993) Asset bubbles and endogenous growth. J Monet Econ 31(1):3-19

[20] Guiso L, Haliassos M, Jappelli T (2003) Household stockholding in Europe: Where do we stand and where do we go? Econ Policy 36:123-164

[21] Guiso L, Jappelli T (2005) Awareness and stock market participation. Rev Finance 9 (14): 537-567

[22] Guiso L, Sapienza P, Zingales L (2008) Trusting the stock market. J Finance 63(6):2557-2600

[23] Haliassos M, Bertaut C (1995) Why do so few hold stocks? Econ J 105(432):1110-1129

[24] Hansen G, İmrohoroğlu S (2016) Fiscal reform and government debt in Japan: A Neoclassical Perspective. Rev Econ Dyn 21:201-224

[25] Hong H, Kubik J, Stein J (2004) Social interaction and stock market participation. J Finance 59(1):137-163

[26] Jappelli T, Padula M (2013) Investment in financial literacy and saving decisions. J Bank Finance 37(8):2779-2792

[27] Kimball M, Shumway T (2006) Investor sophistication, and the participation, home-bias, diversification, and employer stock puzzles. mimeo, University of Michigan

[28] King IP (1992) Endogenous growth and government debt. South Econ J 59(1):15-21

[29] Lusardi A, Michaud P, Mitchell S (2017) Optimal financial knowledge and wealth inequality. J Polit Econ 125(2):431-477

[30] Mankiw GN, Zeldes SP (1991) The consumption of stockholders and nonstockholders. J Financ Econ 29(1):97-112

[31] Motoyama T (2019) Sustainability of public debt under physical and human capital accumulation in an overlapping generations model. J Econ (Zeitschrift fur Nationalokonomie) 127(1):19-45

[32] Ono T (2019) Fiscal rules in a monetary economy: implications for growth and welfare. J Public Econ Theory 22(1):191-219

[33] Song Z, Storesletten K, Zilibotti F (2012) Rotten parents and disciplined children: a politico-economic theory of public expenditure and debt. Econometrica 80(6):2785-2803

[34] Spataro L, Corsini L (2017) Endogenous financial literacy, saving, and stock market participation. FinanzArchiv 73(2):135-162

[35] Tirole J (1985) Asset bubbles and overlapping generations. Econometrica 53(6):1499-1528

[36] Thomas A, Spataro L (2018) Financial literacy, human capital and stock market participation in Europe. J Fam Econ Issues 39(4):532-550

[37] Trabandt M, Uhlig H (2011) The Laffer curve revisited. J Monet Econ 58(4):305-327

[38] van Rooij M, Lusardi A, Alessie R (2011) Financial literacy and stock market participation. J Financ Econ 101(2):449-472

[39] Yakita A (2008) Sustainability of public debt, public capital formation, and endogenous growth in an overlapping generations setting. J Public Econ 92(3-4):897-914

[40] Yakita A (2014) Involuntary unemployment and sustainability of bond-financed fiscal deficit. J Macroecon 41:79-93

[41] Yoong J (2011) Financial literacy and stock market participation: Evidence from the RAND American Life Panel. In: Mitchell OS, Lusardi A (eds) Financial Literacy: Implications for Retirement Security and the Financial Marketplace. Oxford University Press, New York, pp 76-97

Appendix

A Proof of $\Phi'(\sigma_t^*) < 0$

Calculating $\Phi'(\sigma_t^*)$ in (6), we have

$$\Phi'(\sigma_t^*) = -(1-\phi)^{\frac{\alpha}{1-\alpha}} \left[\phi' \frac{1}{1-\alpha} \left(1 - \frac{\sigma_t^*}{\phi} \right) + (1-\phi) \frac{\phi - \sigma_t^* \phi'}{\phi^2} \right]. \tag{A.1}$$

From (2b) and (3), the sign of the first term in square brackets is positive. $\phi - \sigma_t^* \phi'$ in the second term in square brackets can be rewritten as $\phi - \sigma_t^* \phi' = \sigma_t^* \left(\frac{\phi}{\sigma_t^*} - \frac{d\phi}{d\sigma_t^*} \right)$. From Figure 2, we can confirm that the sign of $\phi - \sigma_t^* \phi'$ is also positive. Hence, $\Phi'(\sigma_t^*) < 0$ holds.

B Derivation of (28)

From (25a) the following holds.

$$\frac{db_u^*}{d\sigma} = (1-e) A \left(1 - \frac{d\sigma_u^*}{d\sigma} \right), \quad \frac{db_u^*}{d\sigma} \gtreqless 0 \Leftrightarrow \frac{d\sigma_u^*}{d\sigma} \lesseqgtr 1. \tag{B.1}$$

Substituting (26a) into (B.1) and arranging, we have

$$\frac{db_u^*}{d\sigma} \gtreqless 0 \quad \Longleftrightarrow \quad A \left[\phi(\sigma_u^*) - \phi(\underline{\sigma}) \right] \lesseqgtr -Q'(\sigma_u^*). \tag{B.2}$$

On the other hand, when σ_i changes uniformly in period t in the unstable BGP, its effect on σ_t^* (i.e., the threshold value in period t) can be calculated from (18) as follows:

$$\frac{\partial \sigma_t^*}{\partial \sigma} = 1 \tag{B.3}$$

Therefore, from (14) and (B.3), the effects of the uniform change in σ_i in period t on the economic growth rate and the government bond yield in the same period t are as follows:

$$\begin{aligned}\frac{dG_{u,t}^*}{d\sigma} &= \frac{\partial \sigma_t^*}{\partial \sigma} \times \left.\frac{\partial G}{\partial \sigma_t^*}\right|_{\sigma_t^*=\sigma_u^*} + \left.\frac{\partial G}{\partial \sigma}\right|_{\sigma_t^*=\sigma_u^*} \\ &= A\left[1-\phi\left(\sigma_u^*\right)-e\right] - A\left[1-\phi\left(\underline{\sigma}\right)-e\right] \\ &= -A\left[\phi\left(\sigma_u^*\right)-\phi\left(\underline{\sigma}\right)\right](<0),\end{aligned} \tag{B.4}$$

$$\frac{dQ_{u,t}^*}{d\sigma} = \frac{\partial \sigma_t^*}{\partial \sigma} \times \left.\frac{\partial Q}{\partial \sigma_t^*}\right|_{\sigma_t^*=\sigma_u^*} = Q'\left(\sigma_u^*\right)(<0). \tag{B.5}$$

Substituting (B.4) and (B.5) into (B.2), we obtain (28).

Australia	Canada	France	Germany	Italy	Japan	Spain	Sweden	UK	US
26.1%	22.2%	22.2%	25.4%	33.1%	23.6%	31.3%	34.2%	7.7%	19.0%

Table 1: Percentage of outstanding domestic government bonds held by private banks in 2020 (Source: "Sovereign Debt Investor Base for Advanced Economies" built based on the methodology of Arslanalp and Tsuda (2014))

Parameter	α	γ	Γ	δ	e	η	$\bar{\sigma}$	$\underline{\sigma}$
Value	0.69	0.37	8.76	1	0.8	1.69	0.07	0.04

Table 2: The benchmark numerical settings of parameters

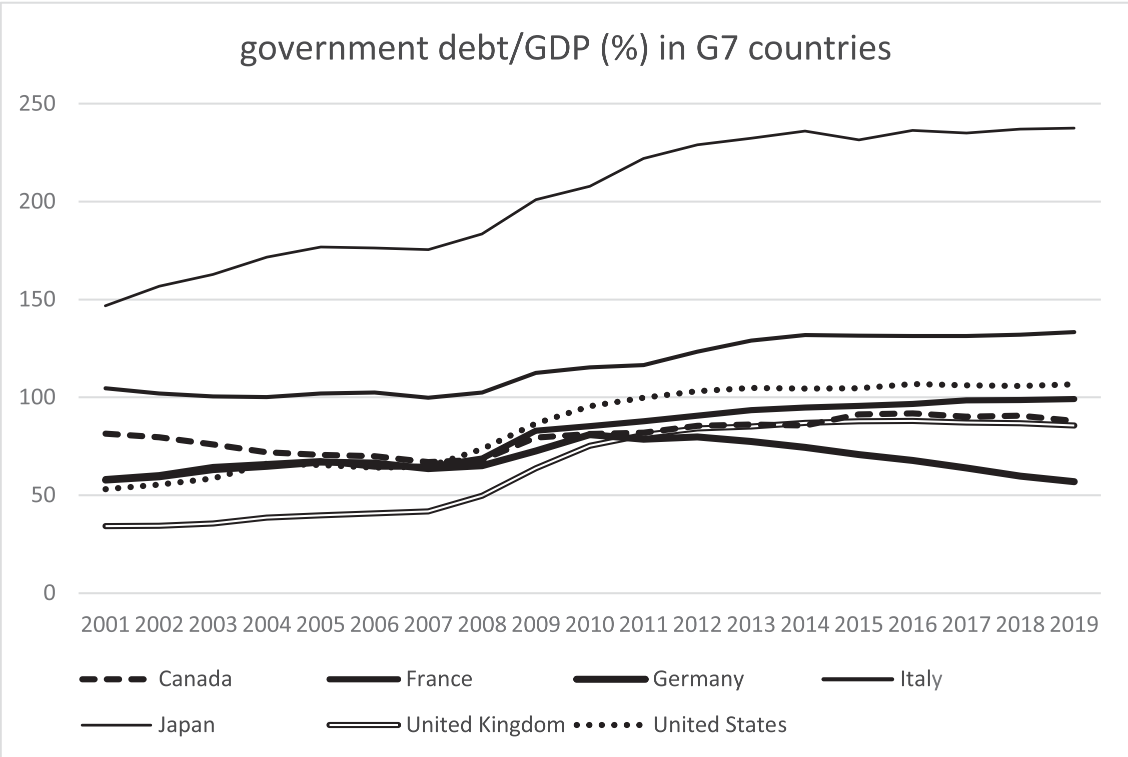

Figure 1: Government debt/GDP (%) in G7 countries (Source: IMF database)

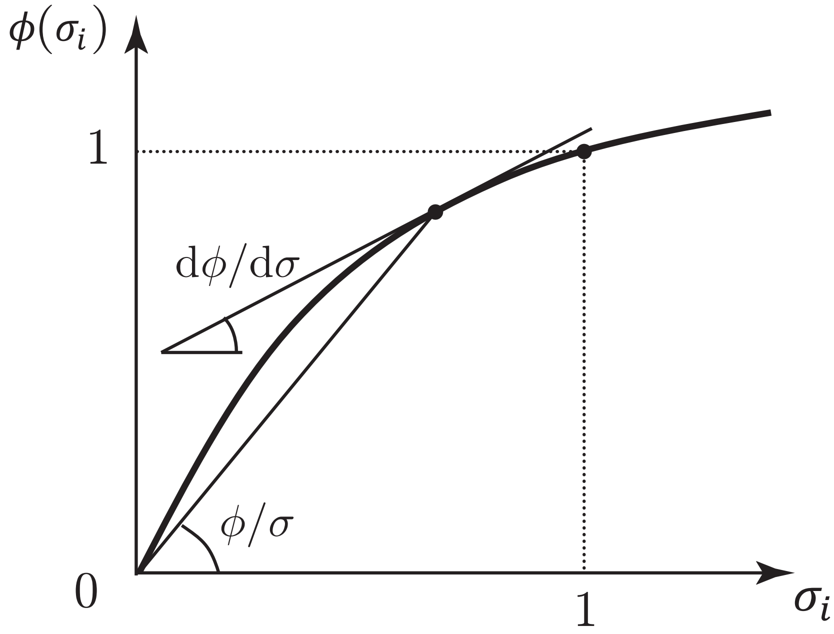

Figure 2: The function $\phi(\sigma_i)$

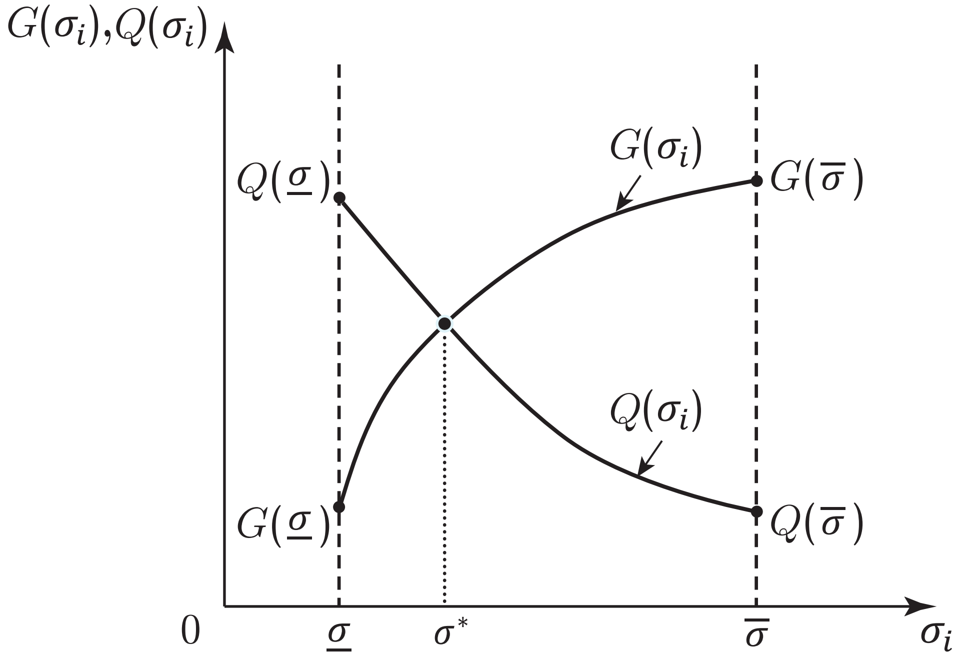

Figure 3: Existence of the BGP with $\underline{\sigma} < \sigma^* < \bar{\sigma}$

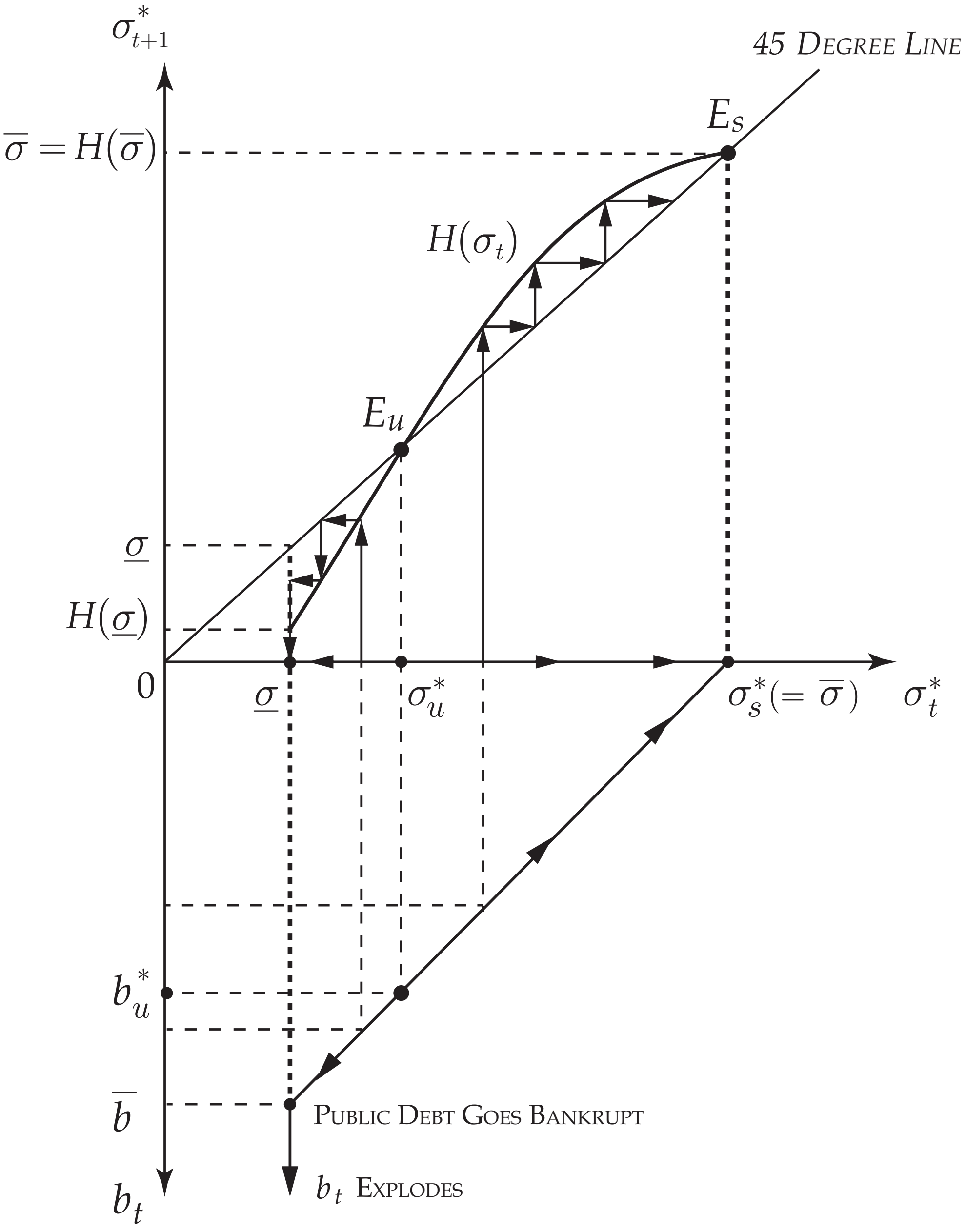

Figure 4: The function $H(\sigma^*_t)$ (the upper half) and $b_t = (1-e)\,A\,(\bar{\sigma} - \sigma^*_t)$ (the lower half)

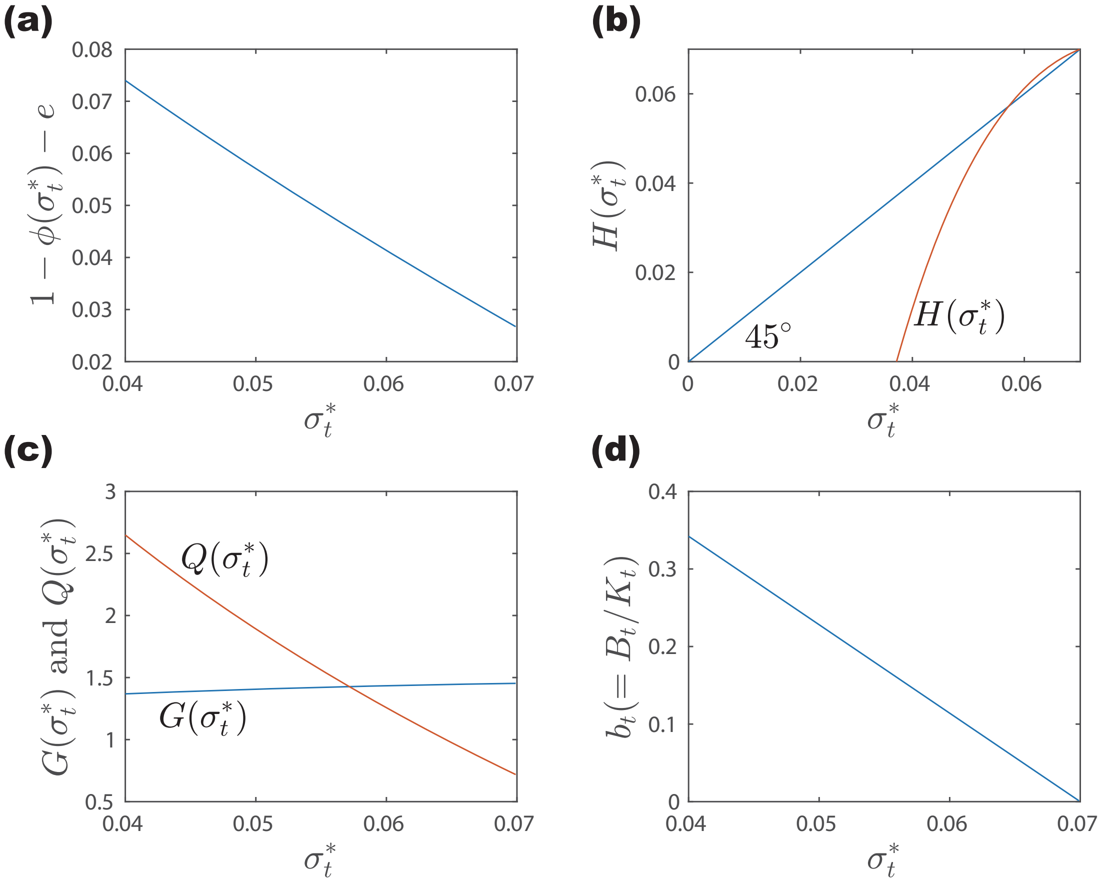

Figure 5: The relationship between σ_t^* and $1-\phi\left(\sigma_t^*\right)-e$, $G\left(\sigma_t^*\right)$, $Q\left(\sigma_t^*\right)$, $H\left(\sigma_t^*\right)$, and b_t

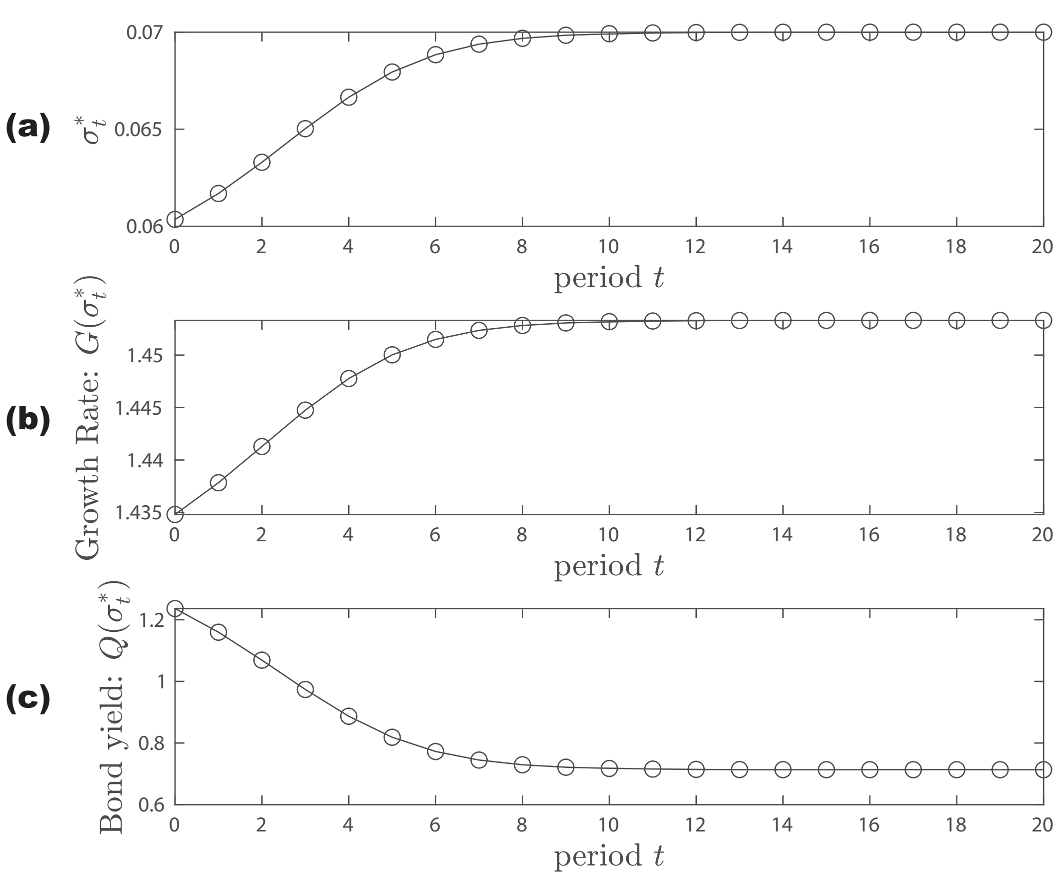

Figure 6: Time paths of σ_t^*, $G(\sigma_t^*)$, and $Q(\sigma_t^*)$: benchmark case

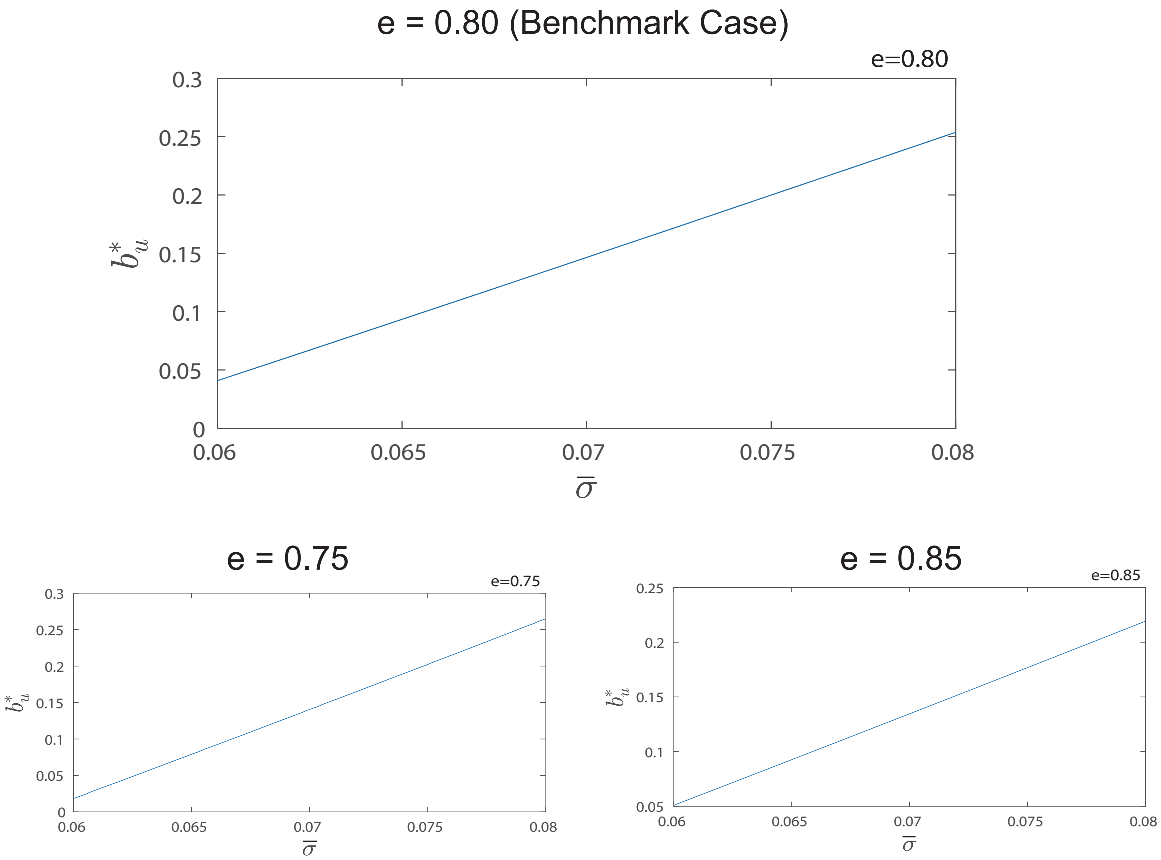

Figure 7: The relationship between $\bar{\sigma}$ and b_u^*

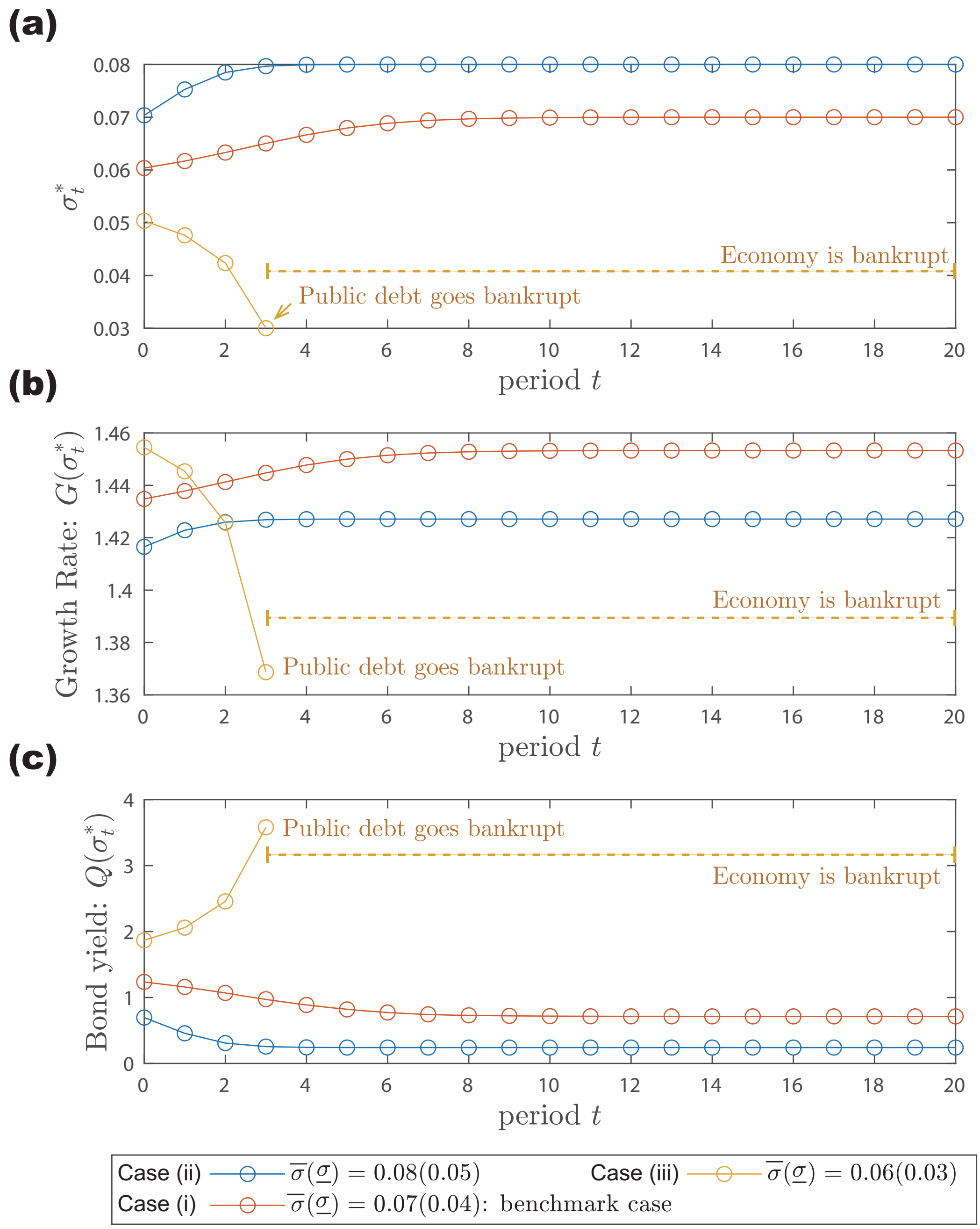

Figure 8: The relationship between $\bar{\sigma}$ and time paths of σ_t^*, $G(\sigma_t^*)$, and $Q(\sigma_t^*)$

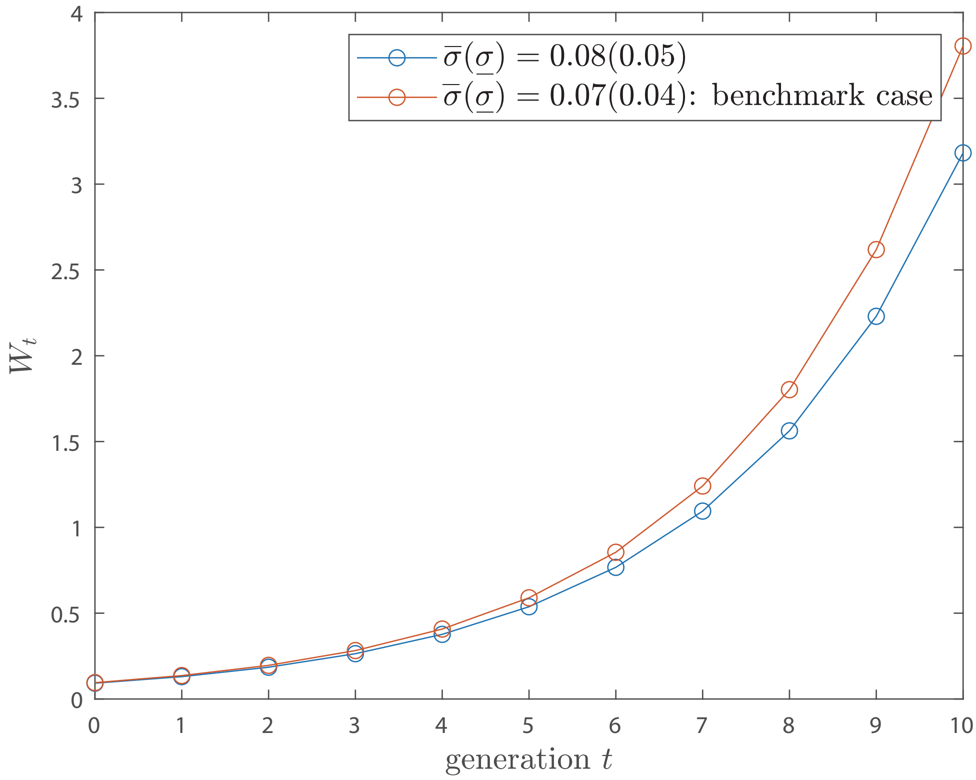

Figure 9: Welfare of each generation

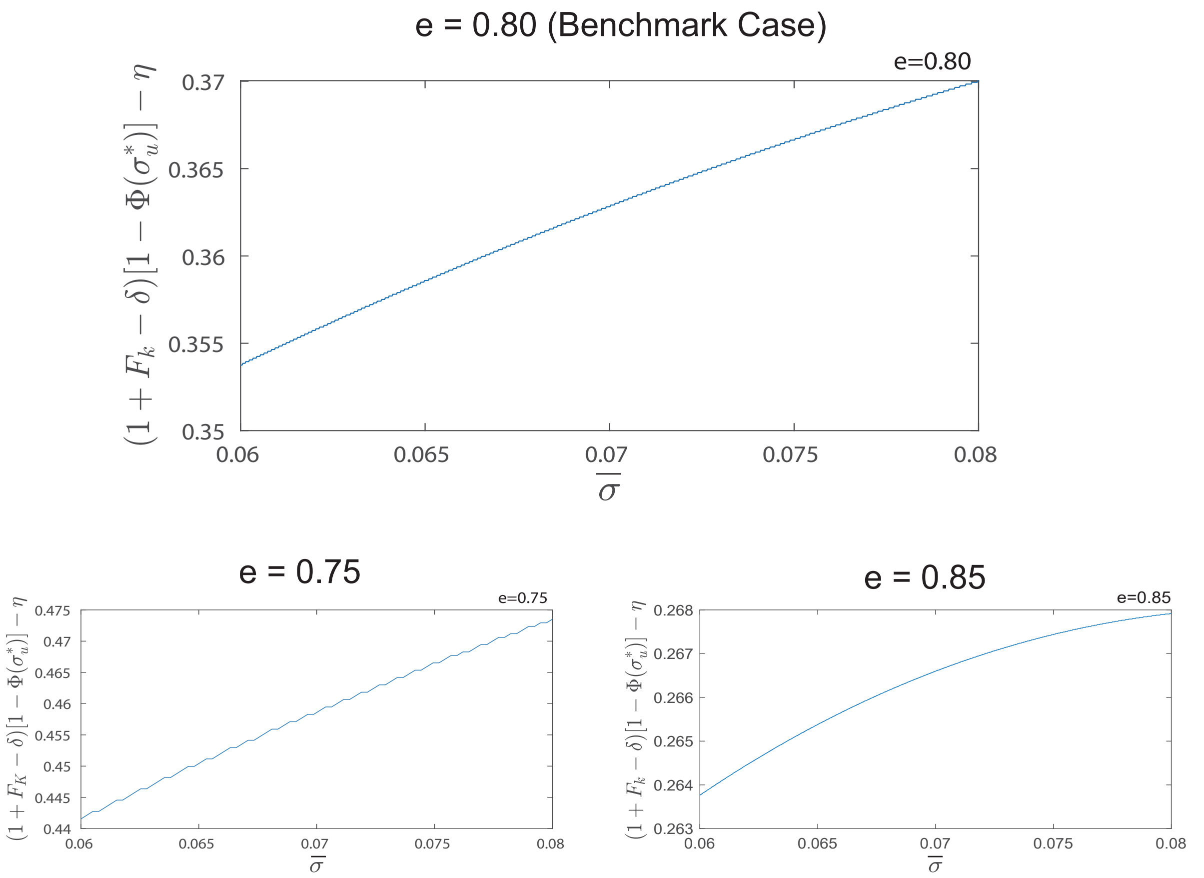

Figure 10: The relationship between $\bar{\sigma}$ and $(1 + F_K - \delta)\left[1 - \Phi\left(\sigma_u^*\right)\right] - \eta$ ($\delta = 1$ in all cases)

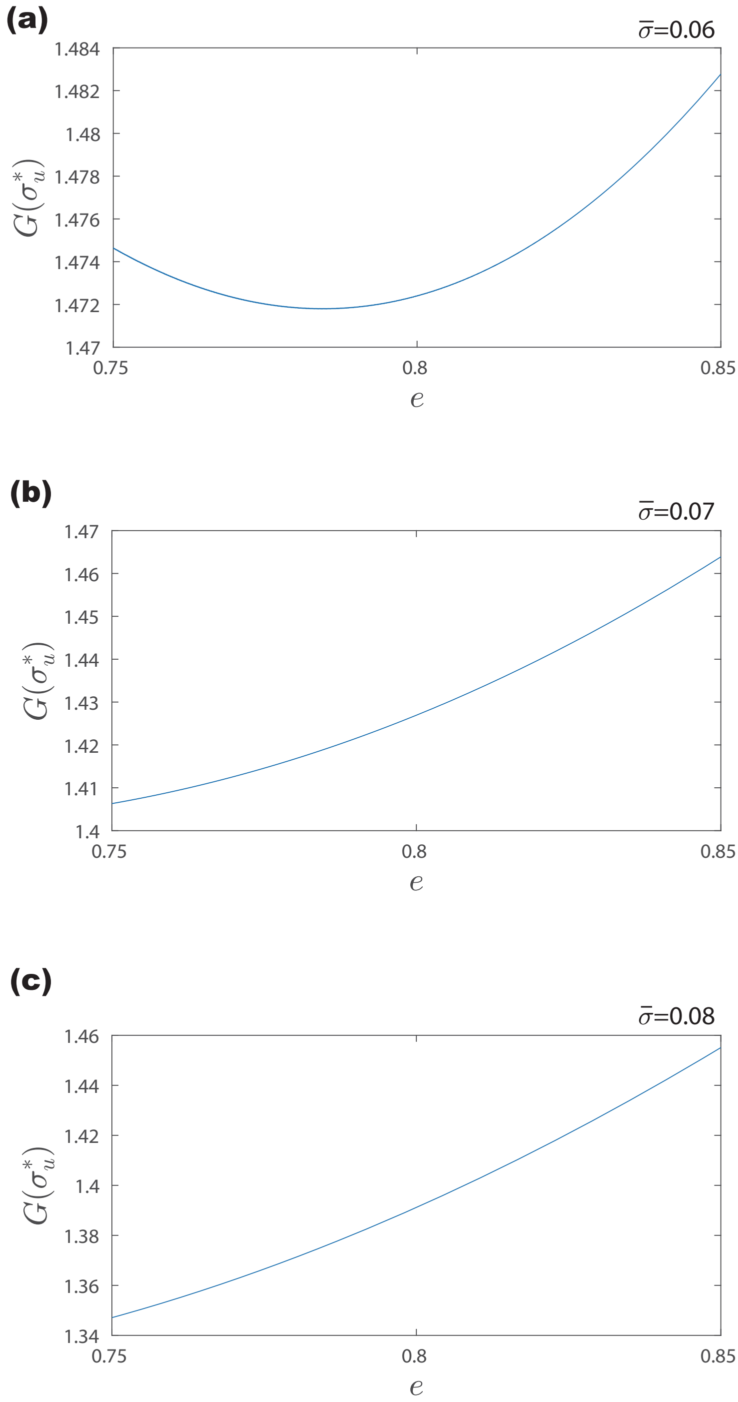

Figure 11: The relationship between e and $G\left(\sigma_u^*\right)$

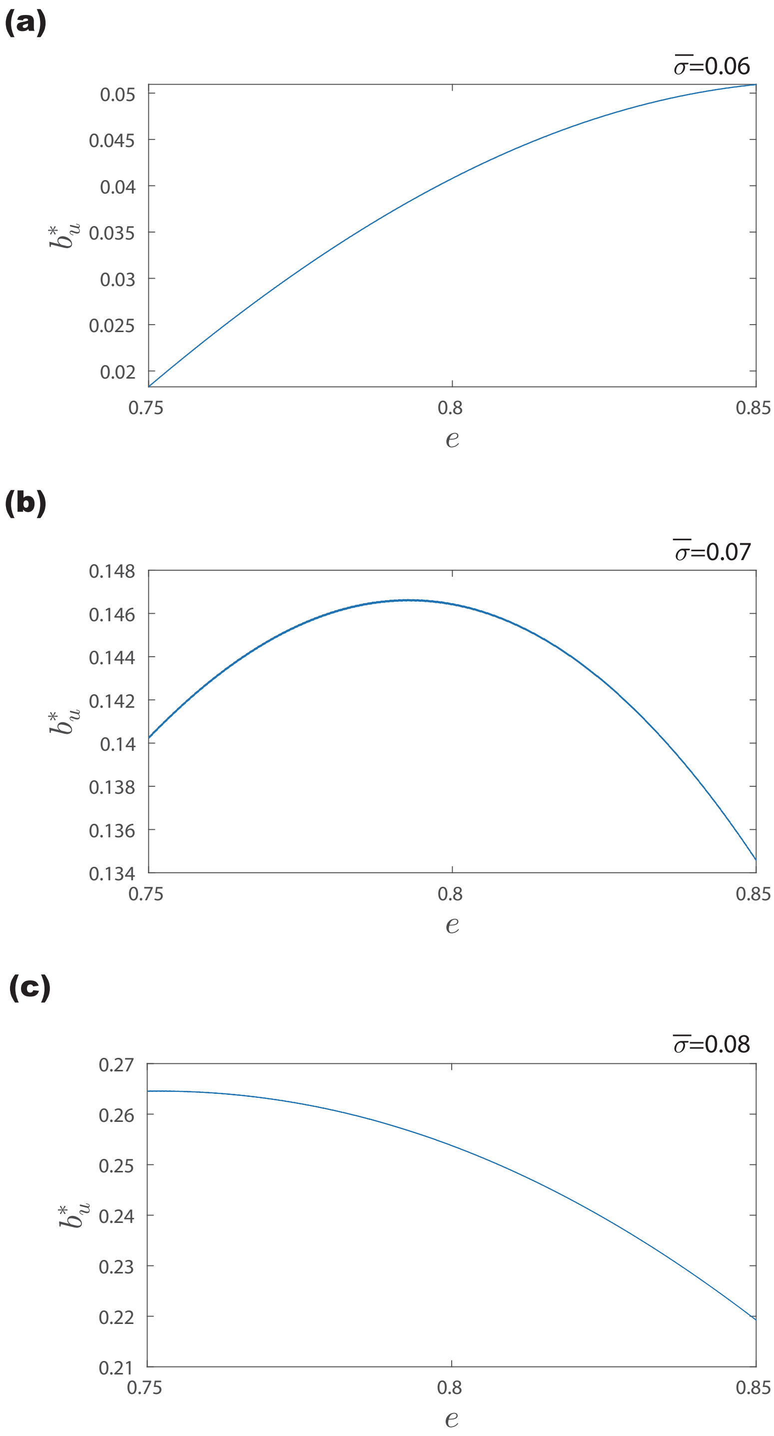

Figure 12: The relationship between e and b_u^*