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Tax Incidence in DSGE Model†

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Abstract

Our paper presents an examination of how an increase in consumption tax affects tax incidence with Dynamic Stochastic General Equilibrium (DSGE) model. Results show that an increase in consumption tax increases the price index level and wage level and decreases consumption, output, and the real interest rate. In addition to these results, our paper presents derivation of results related to inequality. Wage inequality between low-skilled and high-skilled labor shrinks in the short run. However, in the medium run, wage inequality rises. The labor income share increases in the short and medium run. However, the capital income share decreases.

Keywords: Consumption tax, DSGE model, Inequality, Tax incidence
JEL Classifications: E62, H22

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1. Introduction

Our paper presents examination of the effects of an increase in a consumption tax to rebuild fiscal systems with a Dynamic Stochastic General Equilibrium (DSGE) model. A DSGE model can show tax incidence in a transition pass, which cannot be shown clearly by an analytical model. Moreover, our manuscript uses Markov chain Monte Carlo methods (MCMC) to estimate parameters appropriately.

In Japan, the ratio of public debt to gross domestic product (GDP) is greater than 200%. An increase in tax revenue seems necessary to reduce public debt needs. However, consumption taxes present difficulties such as regressivity, even if the effect of a consumption tax on economic performance is less than that of an income tax.

Our paper sets a standard DSGE model including a consumption tax and two types of labor (High-skill labor and Low-skill labor) to consider inequality. These analyses examine the tax incidence of a consumption tax and derive the following results. First, an increase in a consumption tax raises the price index level and reduces consumption and output. Second, because of a decrease in an opportunity cost of leisure, the wage level increases because an increase in a consumption tax reduces the labor supply. Third, the real interest rate decreases because the marginal productivity of capital decreases as a result of decreased labor input. Fourth, the wage inequality between low-skill and high-skill labor shrinks in the short run. However, wage inequality rises in the medium term. Fifth, considering capital income and labor income shares, the labor income share rises and the capital income share declines in the short and medium run.

Some related reports exist the relevant literature. Hayashi (1995) examines tax incidence empirically. However, when considering tax incidence theoretically, it is necessary to set household optimization. Heer and Trede (2003), Nishiyama and Smetters (2005), and Lehmus (2011) set the DSGE model with household optimization and examine tax incidence for some economically development countries. Sakuragawa and Hosono (2011) examine fiscal sustainability in the DSGE Model. However, these studies do not consider income inequality. The problems of consumption tax are attributed to the income inequality because of its regressivity. Therefore, our paper sets the DSGE model with labor of two types: highly skilled labor and low skilled labor. Hara, Katayama and Kato (2014) set the DSGE model with labor of two types. However, they do not examine the effect of tax reform on deriving the tax incidence. Doi (2010) derives the tax incidence of corporation tax, but the inequality between laborers is not considered.

The motivation for the analyses presented in our manuscript is appropriate. The results of these analyses are expected to hold many contributions for this field. Moreover, the parameters are appropriately set by Bayesian statistics. Recent reports of the literature such as those by Smets and Wouters (2007) and Benchimol and Forcans (2012) described studies using Bayesian statistics to estimate parameters. In the DSGE model, few reports of the literature describe studies that use Bayesian statistics except for Hirose and Naganuma (2010) and others.

The remainder of this paper comprises the following. Section 2 sets the model. Section 3 derives the equilibrium. Section 4 sets the parameters. Section 5 presents examination of the results of the simulation.

2. Model

There exist agents of three types: households, firms, and government.

2.1 Household

Households exist for an infinite time and obtain utility from consumption and the money stock. The population size is assumed as unity and no population growth. Our paper assumes the following CRRA form utility as

$$u_t = \frac{c_t^{1-\theta}}{1-\theta} + \frac{m_t^{1-\mu}}{1-\mu} - \frac{l_t^{1+\kappa}}{1+\kappa}. \tag{1}$$

In that equation, c_t, m_t, and l_t respectively denote consumption, the money stock, and labor supply time. Each household has a unit of time. Then the leisure time is $1 - l_t$. β is the discount rate ($0 < \beta < 1$) and θ, μ, κ are parameters showing the level of relative risk aversion. t represents time.

Each household supplies labor to obtain the labor income and to have capital stock to obtain the capital income. However, money stock has no interest. Household allocates the income to money m_t, bond b_t, investment I_t to have capital stock K_t, and consumption c_t.

This model includes labor of two types: high-skill labor (Type H) and low-skill labor (Type L). Type H labor obtains wage rate w_t^H. Type L labor obtains wage rate w_t^L.

The budget constraint of type H can be shown as follows. Superscripts h and l respectively denote type H and type L variables. Lack of a superscript denotes variables that are applicable for both type H and type L.

$$\begin{aligned} m_t + b_t + (1+\tau_c)c_t^h + I_t \\ = \frac{1}{1+\pi_t}\left[(1+i_t)b_{t-1} + m_{t-1}\right] + \varphi_t + (1-\tau)a^h w_t^h l_t^h + r_t K_{t-1}. \end{aligned} \tag{2}$$

The budget constraint of type L is shown as follows. Type L labor allocates all labor income to consumption: our manuscript assumes no saving about type L.

$$(1+\tau_c)c_t^l = (1-\tau)a^l w_t^l l_t^l. \tag{3}$$

In that equation, a^h and a^l respectively denote the labor productivity of type H and type L labor; $a^h > a^l > 0$ is assumed. b_t denotes the stock of no risk asset, which brings about the interest. The nominal interest rate is i_t. Households have capital stock and lend to firms to obtain capital income. The real interest rate is r_t. Moreover, the household of type H has firms and obtains monopolistic profit φ_t. π_t denotes the inflation rate, which is given as $1+\pi_t = \frac{p_t}{p_{t-1}}$. p_t denotes the price index level. w_t represents the effective wage rate. Also, τ_c and τ respectively denote the tax rate of consumption and labor income.

The capital stock in t period is formed as

$$K_t = I_t + (1-\delta)K_{t-1} - S\left(\frac{I_t}{I_{t-1}}\right)I_t . \tag{4}$$

We assume $S' > 0, S(1) = S'(1) = 0$. We can set the following Lagrange equation as

$$\begin{aligned} L = {} & E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{{c_t^h}^{1-\theta}}{1-\theta} + \frac{m_t^{1-\mu}}{1-\mu} - \frac{{l_t^h}^{1+\kappa}}{1+\kappa} \right] \\ & + E_0 \sum_{t=0}^{\infty} \lambda_t \left[m_t + b_t + (1+\tau_c)c_t^h + I_t - \frac{1}{1+\pi_t}\left[(1+i_t)b_{t-1} + m_{t-1}\right] - \varphi_t - (1-\tau)a^h w_t^h l_t^h \right. \\ & \left. - r_t K_{t-1} \right] \\ & + E_0 \sum_{t=0}^{\infty} \gamma_t \left[K_t - I_t - (1-\delta)K_{t-1} + S\left(\frac{I_t}{I_{t-1}}\right) I_t \right], \end{aligned} \tag{5}$$

where λ_t and γ_t denote Lagrange multipliers.

The optimization problem for a household of type H is reduced to the following equations.

$$ {c_t^h}^{-\theta} = \beta E_t {c_{t+1}^h}^{-\theta} \frac{1+i_{t+1}}{1+\pi_{t+1}}, \tag{6}$$

$$ m_t^{-\mu} = {c_t^h}^{-\theta} E_t \frac{2+i_{t+1}}{1+i_{t+1}}, \tag{7}$$

$$ E_t\big(r_{t+1} + q_{t+1}(1-\delta)\big) = q_t E_t \frac{1+i_{t+1}}{1+\pi_{t+1}}, \tag{8}$$

where $q_t = \frac{\gamma_t}{\lambda_t}$.

2.2 Firms

This model includes firms of two types: firms that produce final goods by inputting intermediate goods and firms that produce intermediate goods.

2.2.1 Final Goods Production

It is assumed that final goods are produced in a perfectly competitive market. Then, the product function is shown as presented below:

$$Y_t = \left(\int_0^1 Y_{jt}^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}. \tag{9}$$

The firm produces final good Y_t by inputting each intermediate good Y_{jt}. We assume that the intermediate goods firm is distributed between 0 and 1 and aggregate intermediate goods firm is unity. Then, the profit function π_t^f is

$$\pi_t^f = p_t Y_t - \int_0^1 p_{jt} Y_{jt} dj. \tag{10}$$

Therein, p_{jt} denotes the price of j th intermediate goods. The demand function of the j th intermediate good is derived as shown below:

$$Y_{jt} = \left(\frac{p_{jt}}{p_t}\right)^{-\varepsilon} Y_t. \tag{11}$$

2.2 Intermediate Goods Production

Intermediate goods are produced by inputting capital stock and labor. The product function is assumed as follows.

$$Y_{jt} = K_{jt}^{\alpha}\left({N_{jt}^h}^{\zeta} {N_{jt}^l}^{1-\zeta}\right)^{1-\alpha}. \tag{12}$$

In that equation, K_j denotes capital stock to produce j th intermediate goods. N_{it}^h and N_{jt}^l respectively denote type H and type L labor input to produce j th intermediate goods. Our paper assumes that the labor share of type H and type L are, respectively, $\nu, 1-\nu$. Then, each labor input is

$$N_{jt}^h = \nu a^h l_t^h. \tag{13}$$

$$N_{jt}^l = (1-\nu) a^l l_t^l. \tag{14}$$

Now, we can consider the following Lagrange equation of cost minimization as follows.

$$M = w_{jt}^h N_{jt}^h + w_{jt}^l N_{jt}^l + r_{jt} K_{jt} + \omega_{jt}\left(Y_{jt} - K_{jt}^{\alpha}\left({N_{jt}^h}^{\zeta} {N_{jt}^l}^{1-\zeta}\right)^{1-\alpha}\right), \tag{15}$$

Therein, ω_{jt} denotes a Lagrange multiplier. We can obtain the following demand for the input factor as shown below.

$$w_{jt}^h = \omega_{jt}(1-\alpha)\zeta\left(\frac{K_{jt}}{\left(a^h \nu l_{jt}^h\right)^{\zeta}\left(a^l(1-\nu) l_{jt}^l\right)^{1-\zeta}}\right)^{\alpha}\left(\frac{a^l(1-\nu) l_{jt}^l}{a^h \nu l_{jt}^h}\right)^{1-\zeta}. \tag{16}$$

$$w_{jt}^{l} = \mathrm{\omega_{jt}}(1-\alpha)(1-\zeta)\left(\frac{\mathrm{K_{jt}}}{\left(a^{h}vl_{jt}^{h}\right)^{\zeta}\left(a^{l}(1-v)l_{jt}^{l}\right)^{1-\zeta}}\right)^{\alpha}\left(\frac{a^{h}vl_{jt}^{h}}{a^{l}(1-v)l_{jt}^{l}}\right)^{\zeta}. \tag{17}$$

$$r_{jt} = \omega_{jt}\alpha\left(\frac{K_{jt}}{\left(a^{h}vl_{jt}^{h}\right)^{\zeta}\left(a^{l}(1-v)l_{jt}^{l}\right)^{1-\zeta}}\right)^{\alpha-1}. \tag{18}$$

Considering the constant returns to scale product function and (16)--(18), total cost C is shown as follows.

$$C = w_{jt}^{h}N_{jt}^{h} + w_{jt}^{l}N_{jt}^{l} + r_{jt}K_{jt} = \omega_{jt}Y_{jt}. \tag{19}$$

Lagrange multiplier ω_{jt} denotes the marginal cost. Considering (11) and (19), the profit function of intermediate goods firm are shown as

$$\pi_{jt} = \frac{p_{jt}}{p_t}\left(\frac{p_{jt}}{p_t}\right)^{-\varepsilon}Y_t - \omega_{jt}\left(\frac{p_{jt}}{p_t}\right)^{-\varepsilon}Y_t. \tag{20}$$

Profit maximization is reduced to the following equation.

$$\omega_{jt} = \frac{\varepsilon-1}{\varepsilon}\frac{p_{jt}}{p_t}. \tag{21}$$

2.3. Government

The government levies a labor income tax and a consumption tax for households. The tax revenue is assumed to be expended for non-productive government expenditure.

2.4. Sticky Price

Our paper presents consideration of sticky prices as shown by Calvo (1983). Calvo (1983) assumes that each firm in monopolistic competition cannot set optimal prices to maximize the profit because of some reason. Then, the firm considers the probability of not changing the price level in a future period and sets the price in the present period. Some calculations engender the following New Keynesian Phillips Curve (NKPC) considering homogeneous firms.

$$ln(1+\pi_t) = E_t ln(1+\pi_{t+1}) + \frac{\rho^2}{1-\rho}\left(ln\frac{\varepsilon}{\varepsilon-1} + ln\omega_t\right), \tag{22}$$

In that equation, ρ denotes the probability that the firm can change the price; $1-\rho$ denotes the probability that the firm cannot change the price. Linearization of (22) engenders

$$\tilde{\pi}_t = E_t\tilde{\pi}_{t+1} + \frac{\rho^2}{1-\rho}\widehat{\omega}_t, \tag{23}$$

where $\tilde{\pi}_t = \pi_t - \pi$ and $\widehat{\omega}_t = \frac{\omega_t-\omega}{\omega}$. Actually, π and ω denote the inflation rate and the marginal cost from the steady state. Hereinafter $\hat{z}_t$ denotes the rate of change of z_t

from the steady state. Furthermore, $\tilde{z}_t$ the difference of z_t from the steady state.

2.5. Monetary Policy

Based on the Taylor rule, monetary policy is provided as

$$\tilde{\imath}_t = \chi\tilde{\imath}_{t-1} + (1-\chi)\{\phi_1 E_t\tilde{\pi}_{t+1} + \phi_2\hat{y}_t\}. \tag{24}$$

The nominal interest rate in period t depends on the nominal interest rate in period $t-1$, the expectation of inflation in period *t+1*, and the output gap in period *t*.

3. Equilibrium

This section presents derivation of the equilibrium and linear model. Considering aggregate consumption as $c_t = \nu c_t^h + (1-\nu)c_t^l$, we obtain[1]

$$\hat{C}_t = \frac{\nu c^h}{C}\hat{c}_t^h + \frac{(1-\nu)c^l}{C}\hat{c}_t^l, \tag{25}$$

where

$$\hat{c}_t^h = E_t\hat{c}_{t+1}^h - \frac{1}{\theta}E_t\tilde{\imath}_{t+1} + \frac{1}{\theta}E_t\tilde{\pi}_{t+1}, \tag{26}$$

$$\hat{c}_t^l = \widehat{w}_t^l + \hat{l}_t^l - \tilde{\tau} - \tilde{\tau}_c. \tag{27}$$

Labor supply is given as

$$\widehat{w}_t^h = \kappa\hat{l}_t^h + \tilde{\tau}_t + \theta\hat{c}_t^h + \tilde{\tau}_c, \tag{28}$$

$$\widehat{w}_t^l = \kappa\hat{l}_t^l + \tilde{\tau}_t + \theta\hat{c}_t^l + \tilde{\tau}_c. \tag{29}$$

Tobin's q dynamics follow as

$$E_t\hat{q}_{t+1} = \frac{1}{1-\delta}\left(\frac{1+i}{1+\pi}\left(\hat{q}_t + E_t(\tilde{\imath}_{t+1} - \tilde{\pi}_{t+1})\right) - \frac{r}{q}\hat{r}_t\right). \tag{30}$$

The investment dynamics can be expressed as

$$\hat{I}_t = \frac{1+i}{2+i+\pi}\hat{I}_{t-1} + \frac{1+i}{2+i+\pi}E_t\hat{I}_{t+1} + \frac{1+i}{(2+i+\pi)S''(1)}\hat{q}_t\ . \tag{31}$$

The expression of the capital stock dynamics is

$$\widehat{K}_t = \delta\hat{I}_t + (1-\delta)\widehat{K}_{t-1}. \tag{32}$$

Market clearing conditions of final goods market are shown as follows.

$$\hat{Y}_t = \frac{C}{Y}\hat{C}_t + \frac{I}{Y}\hat{I}_t. \tag{33}$$

The rates of change of factor prices are shown as follows,

$$\widehat{w}_t^h = \widehat{\omega}_t + \alpha\widehat{K}_t - (\alpha\zeta + 1 - \zeta)\hat{l}_t^h + (1-\alpha)(1-\zeta)\hat{l}_t^l, \tag{34}$$

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[1] We consider $\tilde{\tau} = \tilde{\tau}_c$=0 for type L labor because the government provides a redistribution policy to exempt income tax burden and provides benefit for low income family for regressivity of the consumption tax in OECD countries.

$$\widehat{w}_t^l = \widehat{\omega}_t + \alpha\widehat{K}_t + (1-\alpha)\zeta\hat{l}_t^h - (\alpha(1-\zeta)+\zeta)\hat{l}_t^l, \quad (35)$$

$$\hat{r}_t = \widehat{\omega}_t + (\alpha-1)\widehat{K}_t + (1-\alpha)\beta\hat{l}_t^h + (1-\alpha)(1-\zeta)\hat{l}_t^l. \quad (36)$$

Linearization of the product function is shown below.

$$\hat{Y}_t = \alpha\widehat{K}_t + (1-\alpha)\zeta\hat{l}_t^h + (1-\alpha)(1-\zeta)\hat{l}_t^l. \quad (37)$$

4. Parameter Setting

Data that we use for estimation are GDP, consumption, investment, the wage rate, the inflation rate, and the interest rate from the first quarter of 1993 to the second quarter in 2016 in Japan.[2] Results of estimation of parameters are shown as follows.

[Insert Table 1 around here.]

[Insert Table 2 around here.]

[Insert Table 3 around here.]

5. Results and Conclusions

In this section, we examine how the consumption tax affects variables such as consumption. We consider the tax reform of an increase in the consumption tax from 5% to 8% and simulate a permanent shock of an increase in the consumption tax. The following figure presents the effects of an impulse shock of an increase in the consumption tax.

[Insert Fig. 1 around here.]

An increase in the consumption tax reduces the aggregate output, consumption, investment, and others. This is an intuitive result. An increase in the consumption tax raises consumer prices and reduces demand for goods.

The main aim of this paper is to examine tax incidence. Our paper presents consideration of the following index to examine tax incidence as shown below. This index refers to Feldstein (1974) and Doi (2010), as

[2] With the HP Filter, we detrend the data of GDP, consumption, investment, and wage rate. We refer to the total cash salary of the "Monthly Labour Survey (more than 30 persons)" in Japan for the wage rate. We consider that the wage of type H is the index of permanent employees and that the wage of type L is the index of part-time employees. We de-mean the data of the inflation rate and the interest rate. To remove the effect of the bubble economy, we use data from 1993.

$$J_h = \frac{w_t^h N_{it}^h}{w_t^h N_{it}^h + w_t^l N_{it}^l + r_t K_{t-1}}, \quad (38)$$

$$J_l = \frac{w_t^l N_{it}^l}{w_t^h N_{it}^h + w_t^l N_{it}^l + r_t K_{t-1}}, \quad (39)$$

$$J_K = \frac{r_t K_{t-1}}{w_t^h N_{it}^h + w_t^l N_{it}^l + r_t K_{t-1}}. \quad (40)$$

where J_K denotes the share of capital income. J_h and J_l respectively denote the share of labor income of type H and that of type L. The results of an impulse shock of consumption tax are shown as the following figures.

[Insert Fig. 2 around here.]

[Insert Fig. 3 around here.]

Fig. 2 and Fig. 3 respectively show the rate of change from the variables in a steady state. As shown in Fig. 2, an increase in the consumption tax reduces the share of capital income and raises the share of labor income. Therefore, although regressivity exists in the consumption tax, the tax burden brought about by regressivity is weakened because the shares of labor income of both type H and type L raises.

As shown in Fig. 3, the rate of increase in wage rate of type L is larger than that of type H in the short run. This result shows that the low income households are better off, compared with high income households. However, in the medium run, such is not the case.

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Appendix

Derivation of (6)--(8)

First-order conditions of Lagrange equation (5) are shown below.

$$\beta^t {c_t^h}^{-\theta} + \lambda_t(1+\tau_c) = 0. \tag{A.1}$$

$$\beta^{t+1} E_t {c_{t+1}^h}^{-\theta} + \lambda_{t+1}(1+\tau_c) = 0. \tag{A.2}$$

$$\beta^t m_t^{-\mu} + \lambda_t - E_t \frac{\lambda_{t+1}}{1+\pi_{t+1}} = 0. \tag{A.3}$$

$$w_t^h = \frac{(1+\tau_c){l_t^h}^{\kappa}}{(1-\tau)a^h {c_t^h}^{-\theta}}. \tag{A.4}$$

$$w_t^l = \frac{(1+\tau_c){l_t^l}^{\kappa}}{(1-\tau)a^l {c_t^l}^{-\theta}}. \tag{A.5}$$

$$1 = q_t\left(1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\left(\frac{I_t}{I_{t-1}}\right)\right) + E_t q_{t+1}\frac{1+\pi_{t+1}}{1+i_{t+1}} S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2. \tag{A.6}$$

$$-\lambda_t r_t - \gamma_t(1-\delta) + \gamma_{t-1} = 0. \tag{A.7}$$

$$\lambda_t - E_t\lambda_{t+1}\frac{1+i_{t+1}}{1+\pi_{t+1}} = 0. \tag{A.8}$$

$$\frac{\partial L}{\partial \lambda_t} = 0. \tag{A.9}$$

$$\frac{\partial L}{\partial \gamma_t} = 0. \tag{A.10}$$

With (A.1), (A.2), and (A.8), we obtain the consumption Euler's equation (6). With (A.1), (A.3), and (A.8), we obtain the marginal rate of substitution about money stock and consumption (7). With (A.7) and (A.8), we obtain the equation of nominal interest rate, real interest rate, and inflation rate (8).

Derivation of (22) and (23)

The optimal price p_t^* is set as follows because of (21) if the firms can set the price to maximize their profit.

$$lnp_t^* = \ln\frac{\varepsilon}{\varepsilon-1} + ln\omega_t + lnp_t. \tag{B.1}$$

Based on Calvo (1983), we consider sticky prices in this model economy. It is assumed that the share of ρ can change the price in t period and share of $1\text{-}\rho$ cannot change the price in period t. Then, the price that the firm cannot change is equal to the price in period $t\text{-}1$. Then, the firms set the following price x_t if there exists uncertainty about price setting, as

$$\ln x_t = \rho lnp_t^* + \rho(1-\rho)E_t lnp_t^* + \cdots \tag{B.2}$$

$$= \rho lnp_t^* + (1-\rho)E_t lnx_{t+1}.$$

Defining $\ln\Delta x_{t+1} = lnx_{t+1} - lnx_t$ and substitute (B.1) into (B.2), we obtain the following equation:

$$E_t ln\Delta x_{t+1} = \rho E_t lnx_{t+1} - \rho\left(ln\frac{\varepsilon}{\varepsilon-1} + ln\omega_t + lnp_t\right). \tag{B.3}$$

However, the price index in period t, p_t, is given as the weighted average of price x_t that the firms can set, and price p_{t-1} that the firms cannot change in period t. Then, the price index in period t is given as

$$lnp_t = \rho lnx_t + (1-\rho)lnp_{t-1}. \tag{B.4}$$

Considering $1+\pi_t = \frac{p_t}{p_{t-1}}$, we obtain the following equation.

$$\rho lnx_t = ln(1+\pi_t) + \rho lnp_{t-1}. \tag{B.5}$$

$$\rho E_t lnx_t = E_t ln(1+\pi_{t+1}) + \rho lnp_t. \tag{B.6}$$

Then, the following equation is obtained.

$$\rho E_t ln\Delta x_{t+1} = E_t ln(1+\pi_{t+1}) - (1-\rho)ln(1+\pi_t). \tag{B.7}$$

With (B.4), (B.6), and (B.7), we obtain (22). Linearization at approximation of the steady state is reduced to (23).

parameter	initial	minimum	maximum	distribution	average
θ	1	0	10	normal distribution	1
δ	0.05	0.01	0.1	uniform distribution	−
α	0.33	0.23	0.43	uniform distribution	−
ρ	0.25	0	0.9999	normal distribution	0.25
χ	0.7	0	0.9999	β distribution	0.6
ϕ_1	2	0	10	normal distribution	2
ϕ_2	0.2	0	10	normal distribution	0.2
S"(1)	0.13	0	10	normal distribution	0.13
ξ	0.9	0	0.9999	β distribution	0.5
κ	2	0	10	normal distribution	2
standard error: NKPC shock	1.5	0	10	inverse γ distribution	1.5
standard error: technology shock	1.5	0	10	inverse γ distribution	1.5
standard error: preference shock	1.5	0	10	inverse γ distribution	1.5
standard error: monetary policy shock	1.5	0	10	inverse γ distribution	1.5
standard error: high skill labor shock	1.5	0	10	inverse γ distribution	1.5
standard error: low skill labor shock	1.5	0	10	inverse γ distribution	1.5
standard error: regulation cost of investment shock	1.5	0	10	inverse γ distribution	1.5
AR(1) Coefficient: NKPC shock	0.5	0	0.9999	β distribution	0.6
AR(1) Coefficient: technology shock	0.5	0	0.9999	β distribution	0.6
AR(1) Coefficient: preference shock	0.5	0	0.9999	β distribution	0.6
AR(1) Coefficient: monetary policy shock	0.5	0	0.9999	β distribution	0.6
AR(1) Coefficient: labor shock	0.5	0	0.9999	β distribution	0.6
AR(1) Coefficient: regulation cost of investment shock	0.5	0	0.9999	β distribution	0.6

Table 1: Prior Distribution of Parameters

(Based on Eguchi (2011), we set S"(1)=0.13.)

paramerer	content	value
C/Y	consumption output ratio at the steady state	0.8
I/Y	investment output ratio at the steady state	0.2
i	nominal interest rate at the steady state	$(1+\pi)/\beta-1$
π	inflation rate at the steady state	0
q	Tobin's q at the steady state	1
r	real interest rate at the steady state	$q(1/\beta+\delta-1)$
C^h/C	consumption of type H ratio	0.9
C^l/C	consumption of type L ratio	0.1

Table 2: Calibrated Parameters

parameter	average	confidence interval	
θ	0.289	0.21	0.37
δ	0.088	0.073	0.1
α	0.247	0.23	0.268
ρ	0.214	0.179	0.252
χ	0.575	0.469	0.679
ϕ_1	8.697	7.286	10
ϕ_2	0.357	0.159	0.545
S"(1)	7.189	5.054	9.889
ξ	0.869	0.746	0.997
κ	0.087	0	0.167
standard error: NKPC shock	7.513	4.977	9.999
standard error: technology shock	0.551	0.426	0.68
standard error: preference shock	2.121	1.465	2.774
standard error: monetary policy shock	0.371	0.269	0.464
standard error: high skill labor shock	0.97	0.844	1.087
standard error: low skill labor shock	0.715	0.609	0.822
standard error: regulation cost of investment shock	4.585	3.926	5.21
AR(1) Coefficient: NKPC shock	0.006	0	0.013
AR(1) Coefficient: technology shock	0.821	0.71	0.931
AR(1) Coefficient: preference shock	0.916	0.886	0.949
AR(1) Coefficient: monetary policy shock	0.901	0.804	0.992
AR(1) Coefficient: labor shock	0.389	0.283	0.492
AR(1) Coefficient: regulation cost of investment shock	0.201	0.057	0.337

Table 3: Posterior Distribution of Parameters

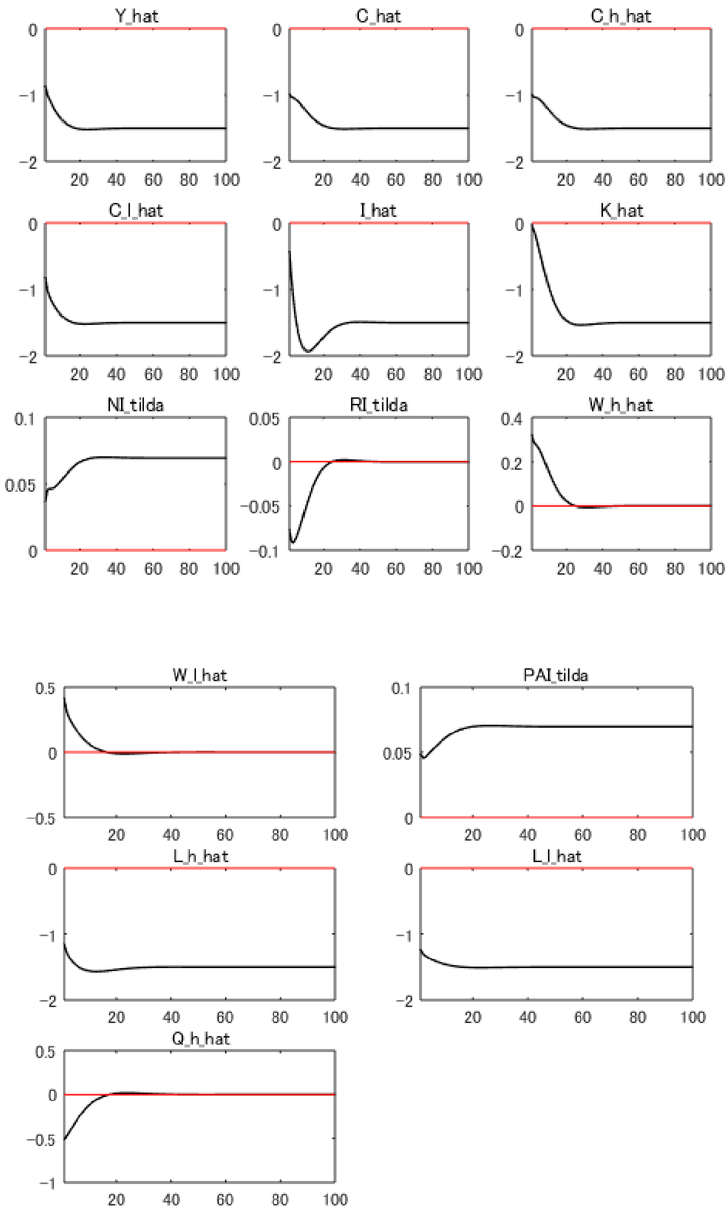

Fig. 1 Impulse Response of an Increase in Consumption Tax.

(Y_hat, C_hat, C_h_hat, C_l_hat, I_hat, K_hat, NI_tilda, RI_tilda, W_h_hat, W_l_hat, PAI_tilda, L_h_hat, L_l_hat and Q_h_hat respectively denote $\hat{y}, \hat{C}, \hat{c}^h, \hat{c}^l, \hat{I}, \hat{K}, \tilde{\imath}, \tilde{r}, \hat{w}^h, \hat{w}^l, \tilde{\pi}, \hat{l}^h, \hat{l}^l, \hat{q}^h$)

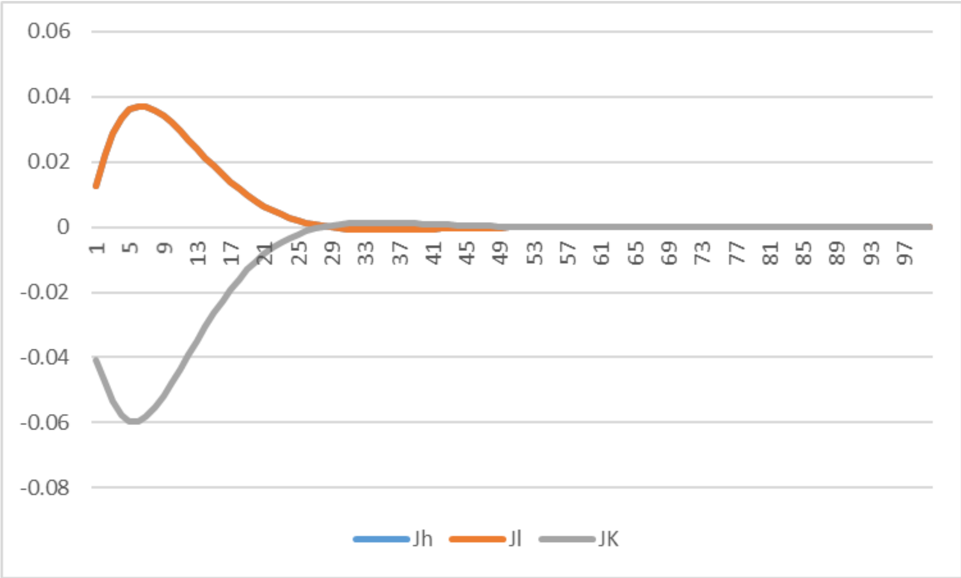

Fig. 2 Impulse Response of an Increase in Consumption Tax for j_h, j_l, j_k .

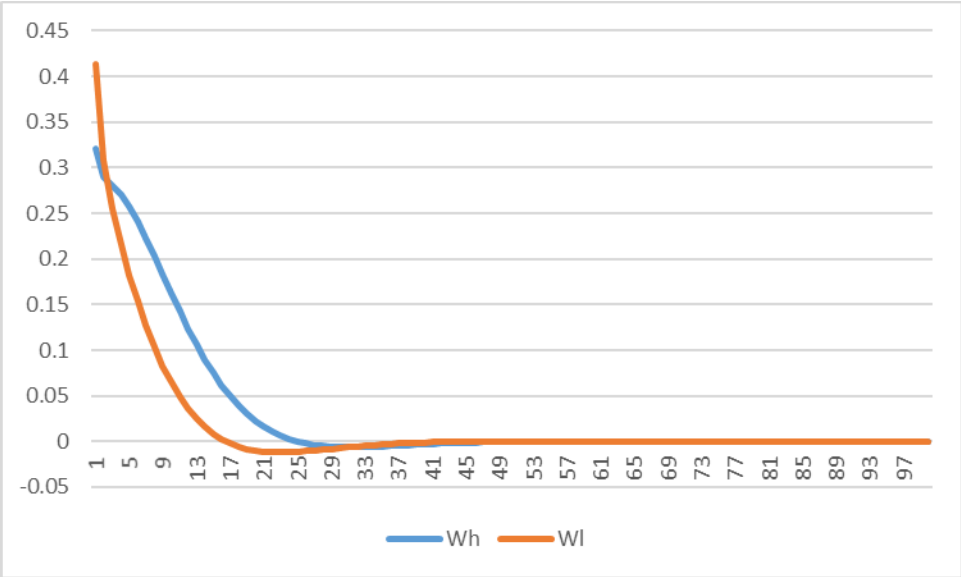

Fig. 3: Impulse Response of an Increase in Consumption Tax for w_h, w_l .