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How Is the Child Allowance to be Financed? By Income Tax or Consumption Tax?*

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Abstract

In economically developed countries, family support policies are provided by the government to raise fertility. Because of an increase in the dependency ratio in many aging societies, governments must pay ever-increasing pension benefits. Therefore, the government must somehow increase the number of younger people to make the pension system sustainable. This paper presents an examination of whether child allowances can raise fertility or not. This issue has been analyzed in numerous earlier studies. However, this paper presents examination of the means used to finance such a child allowance: income taxes and consumption taxes. The different means of taxation exert substantially different effects on fertility.

The results presented in this paper are as follows. First, a child allowance financed by an income tax can not always raise fertility. However, such an allowance financed by a consumption tax can always raise fertility. Second, this paper presents an examination of an optimal tax policy to maximize social welfare. An optimal child allowance and an optimal income transfer from younger people to older people differ according to whether they are financed by an income tax or by a consumption tax.

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1 Introduction

Some OECD countries are experiencing an aging society with low birth rates, as described by Sleebos (2003). The total fertility rate in Japan in 2010 was 1.39. Italy and Germany also suffered a decrease in fertility. In contrast, France and Sweden provide extensive fiscal support for families with policies that have apparently halted the decrease in fertility. Figure 1 shows the correlation between the total fertility rate and fiscal support for families in some OECD countries.

[Insert Fig. 1 around here.]

As Fig. 1 shows, increased fiscal support for families brings about higher fertility than that in countries that do not support families actively.[1] Therefore, it is necessary that the government give a child allowance actively to parents, as is done in France and Sweden, and thereby bear child-care costs publicly and visibly: Japan must increase expenditures for family policies actively to raise the fertility rate in Japan. Macdonald (2006) surveys some earlier papers that describe examinations of family policy. Some earlier papers describe that financial incentives can be effective for increasing fertility (Lutz (1999), Milligan (2002), Laroque and Salanie (2005), Lyssiotou (2012)). In earlier studies, Zhang (1997), Oshio (2001), van Groezen et al. (2003), Yasuoka (2006), van Groezen and Meijdam (2008), Yasuoka and Goto (2011) show that child allowances can raise fertility because of a decrease in the net cost of caring for children. However, Fanti and Gori (2009) show that the child allowance reduces fertility because it prevents capital accumulation by decreasing saving by younger people and by temporarily increasing population growth.[2]

However, these earlier studies do not sufficiently examine how a child allowance is financed. Many of these studies consider an income tax or lump-sum tax as the appropriate means to finance child allowances. As another tool to finance child allowances, we consider a consumption tax. A few studies have been undertaken to examine the effects of a child allowance. Yasuoka and Goto (2011) examine whether a child allowance can raise fertility or not if a consumption tax is used as a tool for finance in a small open economy. Yasuoka (2006) examines whether a child allowance financed by a consumption tax

[1] The child allowance level in France is higher than that in Japan. In Japan, the child allowance is given for children of junior high school age. In France, the child allowance is given for children under 20 years old, except for the first-born child. The amount of the child allowance is 10,000 yen per month in Japan (10,000 yen per month for third-born child of elementary school.). The amount in France is 19,000 yen for the second child (25,000 yen for the third child). In addition to this allowance, children who are older than 11 years old are given a larger allowance (Data: Cabinet Office, Government of Japan (2007):"White Paper on Birthrate-Declining Society").

[2] Originally, Fanti and Gori (2009) report that taxes for children (negative child allowance) can raise fertility. This result is the same as that obtained when the positive child allowance reduces fertility.

can raise fertility or not with numerical examples obtained for a closed economy. Our paper analytically and theoretically examines the effect of a child allowance financed in a closed economy.[3]

Figure 2 shows the national burden rate in some OECD countries.

[Insert Fig. 2 around here.]

The national burden in European countries is about 50% − −60%. In contrast, the national burden in Japan is less than 40%. Therefore, Japan can not provide sufficient fiscal support for families, as Fig. 1 shows. In Japan, the share of older people who are more than 65 years old to the total population is at 23.0% in 2010.[4] and it is expected that the aging society will be progressing steadily. Therefore, it will be increasingly difficult to provide a child allowance financed by income tax because the share of younger people to total population will decrease.

Why must the government restore declining fertility? It must do so because the government is obligated to manage the social security system. Social security systems such as those of health insurance and pension systems must be supported by younger generations. Unless the population of the young generation is sufficiently large, the social security system can not be maintained. Concretely, the government must collect a contribution from younger people to run the pay-as-you-go pension system. If the population of the younger generation is small, then the pension that older people receive under a constant contribution rate would be small, too. The burden of the younger people would become quite large if the government were to fix the pension paid to older people. Therefore, unless the population is sufficiently large, the government can not afford to pay pensions to older people sufficiently and then, can not maintain the system.

[Insert Fig. 3 around here.]

Figure 3 shows government social spending and social spending for people of advanced age. In European countries, public social expenditures are larger than for people in Japan. However, regarding old age social expenditure, that in Japan is nearly same as that in European countries: the share of old age social expenditures to government social spending in Japan is greater than that in European countries.

[3]In a closed economy, we consider capital accumulation, which affects household income, the wage rate, and the interest rate. These changes also affect fertility. The effects of an income tax and consumption tax on capital accumulation differ. In a small open economy, there is substantially no difference between the two tax systems because we do not consider capital accumulation.

[4]Data: A 2012 Declining Birthrate White Paper (2012)

Actually, we must consider the following question. How is optimal social spending for both younger people and older people determined? For a small open economy, van Groezen et al. (2003) examine the optimal child allowance. Yasuoka and Goto (2011) describe how fertility is determined under a small open economy that has adopted a pay-as-you-go pension system. Moreover, they describe how the child allowance affects fertility and the welfare level, how the child allowance is financed, and how both increased fertility by the child allowance and the direct income transfer to older people affect the welfare of older people. Endogenous fertility with a pay-as-you-go pension engenders market failure. Oshio (2001) and van Groezen and Meijdam (2008) set a closed economy model and examine the optimal child allowance. Different from our manuscript is the assumption of income taxation alone. Our paper presents consideration not only of income tax but also of a consumption tax. Yasuoka (2006) examines how the child allowance is financed: with a labor income tax, a capital income tax, and a consumption tax, and derives the result that the child allowance should not be financed by a labor income tax because of a large decrease in social welfare. However, Yasuoka (2006) does not calculate the optimal child allowance, the income transfer from younger people to older people.[5]

The aims of our paper are the following. First, we present an examination of the effect of a child allowance financed by income tax or consumption tax on fertility. Second, our analyses derive an optimal child allowance and an optimal income transfer from younger people to older people. We set an endogenous fertility model with a child allowance and pay-as-you-go pension system in a closed economy. The endogenous fertility models presented by Nishimura and Zhang (1992), Zhang and Zhang (1998) and Oshio and Yasuoka (2009) incorporate children as investment goods.[6] These papers show derivations demonstrating that public pensions decrease fertility because the parent does not need children to care for their life when they are old. However, if children are regarded as consumption goods, then public pensions might increase fertility, as shown in the model by van Groezen et al. (2003) and others.

The results presented in theoretical papers collectively imply a positive correlation between fertility and income (van Groezen et al. (2003) and Fanti and Gori (2009) and others). Galor and Weil (1996)

[5]Some earlier studies examine the pension system to improve social welfare. Hirazawa and Yakita (2009) report their results, showing that the tax cut improves social welfare in endogenous fertility model in pay-as-you-go pension. Lin and Tian (2003) examine whether consumption taxes raise social welfare to increase pension benefits or not in the model with endogenous fertility model considering children as investment goods. However, pension benefits financed by consumption taxes are ambiguous.

[6]Becker and Barro (1988) and Barro and Becker (1989) set the dynamic general equilibrium model with endogenous fertility. Based on Becker and Barro (1988) and Barro and Becker (1989), many papers present examinations of factors determining fertility.

show a negative correlation between fertility and wage income. In contrast, Ahn and Mira (2002) show a positive correlation between fertility and the female labor participation rate in OECD countries. Apps and Rees (2004), Martínez and Iza (2004) and Yasuoka and Miyake (2011) consider the child-care sector and derived a theoretical model that is consistent with the positive correlation. This positive correlation engenders a positive correlation between fertility and income. Galor and Weil (1996) consider a high opportunity cost for child care. Opportunity costs for child-care costs determine the correlation between fertility and income. In fact, van Groezen et al. (2003) and others imply no large opportunity cost and therefore positive correlation between fertility and income. The analyses described in this paper assume an endogenous fertility model, implying a positive correlation.

The results derived from our analyses and presented in this manuscript are as follows. First, a child allowance financed by a consumption tax can always raise fertility, although a child allowance financed by income tax will not always raise fertility. Second, based on how the child allowance is financed, an optimal income transfer from younger people to older people and an optimal child allowance differ between a child allowance financed by an income tax and one financed by a consumption tax.

The remainder of this paper is presented as follows. Section 2 introduces the model. Section 3 presents a description of the equilibrium. Section 4 presents derivation of the first best solution, which maximizes social welfare and optimal tax rate and optimal child allowance to achieve the first-best solution. The final section presents conclusions of this study.

2 The Model

The model economy is based on a two-period (young and old) overlapping generations model. This economy has agents of three types: households, firms, and a government.

2.1 Households

Households experience two periods: young and old. During the young period, each household supplies labor to earn labor income. Households have one unit of time, which is assumed to be supplied for labor inelasticity. Younger people consume and raise children. These analyses assume that it is necessary for households (parents) to invest in child care to have children. During the older period, each household only consumes. The government imposes a payroll tax to provide income transfer to older people. In addition, the government imposes a payroll tax and consumption tax to provide a child allowance. Each household distributes its labor income among child-care goods and other consumption. Consequently, we obtain the following budget constraint.

$$(1+\tau_{ct})c_{1t} + \frac{(1+\tau_{ct+1})c_{2t+1}}{1+r_{t+1}} + (z-q)n_t = (1-\theta-\tau_t)w_t + \frac{p_{t+1}}{1+r_{t+1}}. \tag{1}$$

Therein, τ_{ct} denotes the consumption tax rate in t period $(0 < \tau_{ct} < 1)$. For these analyses, we assume that the government does not impose a consumption tax for child-care goods, and therefore does not affect fertility negatively.[7] Younger people face a payroll tax by tax rate in t period τ_t $(0 < \tau_t < 1)$ to provide a child allowance q and θ $(0 < \theta < 1)$ to provide income transfer to older people. The pension received by older people is p_{t+1}. In addition, w_t and r_{t+1} respectively represent the wage rate in t period and interest rate in $t+1$ period. Furthermore, z denotes the child-care cost per child. Parents are given a child allowance q for a child, which is assumed as $z > q$. In addition, c_{1t}, c_{2t+1} and n_t respectively represent consumption by younger people, older people, and the number of children.

A household's utility function u_t is given as follows:[8]

$$u_t = \alpha \ln c_{1t} + \beta \ln c_{2t+1} + (1-\alpha-\beta)\ln n_t,\ 0 < \alpha, \beta < 1,\ \alpha+\beta < 1. \tag{2}$$

An individual chooses consumption during young and old life c_{1t}, c_{2t+1} and chooses the number of children n_t to maximize lifetime utility (2) subject to the lifetime budget constraint (1). The optimal allocations are determined as

$$c_{1t} = \frac{\alpha}{1+\tau_{ct}}\left((1-\tau_t-\theta)w_t + \frac{p_{t+1}}{1+r_{t+1}}\right), \tag{3}$$

$$c_{2t+1} = \frac{\beta(1+r_{t+1})}{1+\tau_{ct+1}}\left((1-\tau_t-\theta)w_t + \frac{p_{t+1}}{1+r_{t+1}}\right), \tag{4}$$

$$n_t = \frac{1-\alpha-\beta}{z-q}\left((1-\tau_t-\theta)w_t + \frac{p_{t+1}}{1+r_{t+1}}\right). \tag{5}$$

2.2 Firms

The production function of final goods is given as a neoclassical constant-returns-to-scale function,

$$Y_t = AK_t^{\epsilon}N_t^{1-\epsilon},\ 0 < \epsilon < 1, 0 < A, \tag{6}$$

[7] In economically developed countries with a consumption tax rate for necessities, child-care service is zero or discounted by the government. Therefore, our analyses assume a zero tax rate for child-care cost per child z.

[8] This assumption is conventional in the modeling of endogenous fertility: Eckstein and Wolpin (1985), Galor and Weil (1996) and van Groezen et al. (2003), and others.

where Y_t and K_t respectively represent the final goods and capital stock. Moreover, N_t denotes labor input and younger population size. Each firm determines the demand for capital stock and labor to maximize profit. Assuming perfect competition, the wage rate w_t and the interest rate r_t are

$$w_t = A(1-\epsilon)k_t^{\epsilon}, \tag{7}$$

$$1 + r_t = A\epsilon k_t^{\epsilon-1}, \tag{8}$$

where $k_t \equiv \frac{K_t}{N_t}$. The capital stock is assumed to be fully depreciated in a single period.

2.3 Government

The government imposes labor income taxation at a tax rate τ_t and consumption at a tax rate τ_{ct} to provide a child allowance q. In addition, the government imposes labor income taxation at a tax rate θ to provide benefits for older people p_{t+1}. The budget of the child allowance and pension are assumed to be separate. The analyses described in this paper assume a balanced budget in each period.[9] Then, the government budget constraint for pension is presented as $N_{t+1}\theta w_t = N_t p_{t+1}$, i.e.,

$$p_{t+1} = \theta w_t n_t, \tag{9}$$

because of $\frac{N_{t+1}}{N_t} = n_t$. Therein, N_{t+1} denotes the population size of younger people in $t+1$ period. Therefore, in the $t+1$ period, the population size of older people is N_t. The government budget constraint for a child allowance is presented as $N_t\tau_t w_t + \tau_{ct}(N_t c_{1t} + N_{t-1}c_{2t}) = N_t q n_t$, that is,

$$qn_t = \tau_t w_t + \tau_{ct}\left(c_{1t} + \frac{c_{2t}}{n_{t-1}}\right). \tag{10}$$

3 Equilibrium and Policy Effect

In this section, we consider two equilibria: one for the equilibrium with a child allowance financed by income tax and the other for the equilibrium with a child allowance financed by the consumption tax.

3.1 Equilibrium with Income Tax

First, we derive the equilibrium in which the child allowance is financed by income taxation. Considering $\tau_{ct} = 0$, Eq. (10) can be transformed to the following equation

$$qn_t = \tau_t w_t. \tag{11}$$

[9] Many earlier papers assume a balanced budget that is similar to that described in this paper (e.g. van Groezen et al. (2003)). Ono (2003) considers public debt and alleviated the assumption of balanced budget.

In that equation, τ_t is determined to hold. Considering Eqs. (5), (9), and (11), fertility n_t is shown as

$$n_t = \frac{(1-\alpha-\beta)(1-\theta)w_t}{z-(\alpha+\beta)q-\frac{(1-\alpha-\beta)\theta w_{t+1}}{1+r_{t+1}}}. \tag{12}$$

Therein, $z-(\alpha+\beta)q-\frac{(1-\alpha-\beta)\theta w_{t+1}}{1+r_{t+1}}$ must be positive because $n_t > 0$. If this model economy is given as a small open economy with wage rate w_t, w_{t+1} and an interest rate r_{t+1} are fixed, then a child allowance can always raise fertility n_t, as shown by Eq. (12), which is obtained by van Groezen et al. (2003) and Yasuoka and Goto (2011). Our paper is intended to examine the effect of a child allowance in a closed economy for which capital accumulation is considered. The dynamics of capital per capita k_t is represented by the capital market equilibrium condition $K_{t+1} = N_t s_t$, where s_t denotes an individual's saving. Dividing this equation by N_t, then

$$k_{t+1} = \frac{s_t}{n_t}. \tag{13}$$

Substituting an individual saving $s_t = (1-\tau_t-\theta)w_t-(z-q)n_t-c_{1t}$, Eqs.(3), (5), (9), and (11) into (13), we obtain the dynamics equation of k_t.

$$k_{t+1} = \frac{(1-\theta)w_t}{n_t} - \frac{(1-\beta)z}{1-\alpha-\beta} + \frac{\alpha q}{1-\alpha-\beta} \tag{14}$$

We consider the steady state equilibrium which holds $k_{t+1} = k_t$. We define n and k respectively as fertility and the capital per capita in the steady state. Considering Eqs. (7), (8), (12), and (14), we obtain n and k, which hold the following two equations:

$$n = \frac{(1-\alpha-\beta)(1-\theta)A(1-\epsilon)k^{\epsilon}}{z-(\alpha+\beta)q-\frac{\theta(1-\epsilon)(1-\alpha-\beta)}{\epsilon}k}, \tag{15}$$

$$k = \frac{(1-\theta)(1-\epsilon)Ak^{\epsilon}}{n} - \frac{(1-\beta)z}{1-\alpha-\beta} + \frac{\alpha q}{1-\alpha-\beta}. \tag{16}$$

Now, we examine whether a child allowance financed by an income tax can raise fertility or not. Differentiating n and k about q at the approximation of $q = 0$, we obtain the following equations as

$$a_{11}\frac{dn}{dq} + a_{12}\frac{dk}{dq} = (\alpha+\beta)n, \tag{17}$$

$$a_{21}\frac{dn}{dq} + a_{22}\frac{dk}{dq} = \frac{\alpha}{1-\alpha-\beta} \tag{18}$$

where

$$a_{11} = z - \frac{\theta(1-\epsilon)(1-\alpha-\beta)}{\epsilon}k,$$

$$
\begin{aligned}
a_{12} &= -(1-\alpha-\beta)(1-\epsilon)\left((1-\theta)A\epsilon k^{\epsilon-1} + \frac{\theta}{\epsilon}n\right), \\
a_{21} &= (1-\theta)A(1-\epsilon)\frac{k^{\epsilon}}{n^2}, \\
a_{22} &= 1-\epsilon(1-\theta)A(1-\epsilon)\frac{k^{\epsilon-1}}{n}.
\end{aligned}
$$

$\frac{dn}{dq}$ is shown as follows,

$$
\begin{aligned}
\frac{dn}{dq} &= \frac{a_{22}(\alpha+\beta)n - a_{12}\frac{\alpha}{1-\alpha-\beta}}{a_{11}a_{22}-a_{12}a_{21}} \\
&= \frac{\frac{n}{\epsilon}\left((\alpha+\beta)\epsilon + \alpha(1-\epsilon)\theta - \beta\epsilon^2 - (1-\beta)\epsilon(\epsilon+\theta(1-\epsilon))\right)}{a_{11}a_{22}-a_{12}a_{21}}. \qquad (19)
\end{aligned}
$$

Considering child allowance $q = 0$, Eqs. (12) and (14), then the dynamics equation of k_{t+1} are given as

$$
k_{t+1} = \frac{\epsilon\beta z}{(1-\alpha-\beta)(\epsilon+\theta(1-\epsilon))}. \qquad (20)
$$

The sign of the denominator of (19) is positive.[10] Therefore, if the sign of the numerator of (19) is positive, then the sign of $\frac{dn}{dq}$ is positive, so a child allowance can raise fertility. This condition is shown as the following

$$
\beta > \frac{(\epsilon+\theta(1-\epsilon))(\epsilon-\alpha)}{\epsilon(1+\theta(1-\epsilon))}. \qquad (21)
$$

Then, as shown by Eq. (21), the following proposition is established.

Proposition 1 A child allowance financed by an income tax can raise fertility in the steady state if Eq. (21) is held.

This proposition always holds if $\epsilon < \alpha$. With small $\alpha + \beta$, then $1-\alpha-\beta$ becomes large, indicating a high preference for having children. An increase in the child allowance can greatly raise fertility, as shown by (5). We infer that the larger $1-\alpha-\beta$ is, the greater the effect of increasing fertility is. However, an increase in fertility and tax burden prevent capital accumulation per capita. This reduces a wage rate, i.e., a child allowance has the effect of decreasing fertility via a decrease in household's income. As long as Eq. (21) holds, which means that α (to be more than ϵ) or β is large (to hold Eq. (21) even if $\epsilon > \alpha$), i.e., the preference for having children is not large, the latter effect is small, then a child allowance can raise fertility. Based on parametric conditions, Fanti and Gori (2009) derived that the tax for children

[10] See for Appendix for a detailed proof.

(negative child allowance) can raise fertility. Substantially, the result in this paper is the same as that in Fanti and Gori (2009). We can find the result in the model given also by Oshio (2001), Yasuoka (2006) and van Groezen and Meijdam (2008).[11]

3.2 Equilibrium with Consumption Tax

Second, we derive the equilibrium in which a child allowance is financed by consumption taxation. Considering $\tau_t = 0$, Eq. (10) changes to the following equation,

$$qn_t = \tau_{ct}\left(c_{1t} + \frac{c_{2t}}{n_{t-1}}\right). \tag{22}$$

Inserting Eqs. (3), (4), and (5) into (22), we obtain

$$qn_t = \frac{\tau_{ct}}{1+\tau_{ct}}\frac{z-q}{1-\alpha-\beta}\left(\alpha n_t + 1 + r_t\right). \tag{23}$$

Then, fertility n_t is given as

$$n_t = \frac{A(1-\alpha-\beta)(1-\epsilon)(1-\theta)k_t^{\epsilon}}{z-q-\frac{(1-\alpha-\beta)(1-\epsilon)\theta}{\epsilon}k_{t+1}}. \tag{24}$$

The dynamics of capital per capita k_t is given by Eq. (13). Substituting an individual saving $s_t \equiv (1-\theta)w_t - (z-q)n_t - (1+\tau_c)c_{1t}$, Eqs. (3), (5), into (13), we obtain the dynamics equation of k_t.

$$k_{t+1} = \frac{(1-\theta)w_t}{n_t} - \frac{(1-\beta)(z-q)}{1-\alpha-\beta} \tag{25}$$

The capital stock per capita k and fertility n in the steady state are given as the following equations:

$$n = \frac{(1-\alpha-\beta)(1-\theta)A(1-\epsilon)k^{\epsilon}}{z-q-\frac{(1-\alpha-\beta)(1-\epsilon)\theta}{\epsilon}k}, \tag{26}$$

$$k = \frac{(1-\theta)A(1-\epsilon)k^{\epsilon}}{n} - \frac{(1-\beta)(z-q)}{1-\alpha-\beta}. \tag{27}$$

Now, we examine whether a child allowance financed by an consumption tax can raise fertility or not. Differentiating n and k for q at the approximation of $q = 0$, we obtain the following equations:

$$a_{11}\frac{dn}{dq} + a_{12}\frac{dk}{dq} = n, \tag{28}$$

$$a_{21}\frac{dn}{dq} + a_{22}\frac{dk}{dq} = \frac{1-\beta}{1-\alpha-\beta}. \tag{29}$$

[11] As shown by (32), aggregate consumption and child care in the steady state is shown by $c_1 + \frac{c_2}{n} + zn$. The maximization of this aggregate consumption reduces to $1 + r = n$, that is, $\beta = \frac{\epsilon}{1-\epsilon} + \theta$. Then, substituting this equation into (21), we obtain $\alpha > \frac{((1-\theta)(1-\epsilon)-1)\epsilon}{1-\epsilon}$, which is always held. That is, given $1 + r > n$, an increase in child allowance financed by income tax can raise the fertility.

Here, $\frac{dn}{dq}$ is shown as follows,[12]

$$\frac{dn}{dq} = \frac{na_{22} - \frac{1-\beta}{1-\alpha-\beta}a_{12}}{a_{11}a_{22} - a_{12}a_{21}}$$
$$= \frac{n\left((1-\epsilon)(1-(1-\beta)\theta) + \frac{(1-\beta)(1-\epsilon)\theta}{\epsilon}\right)}{a_{11}a_{22} - a_{12}a_{21}} > 0. \tag{30}$$

As long as the denominator $a_{11}a_{22} - a_{12}a_{21} > 0$, which is derived by a locally stable condition, the sign of $\frac{dn}{dq}$ is always positive. A child allowance can always raise fertility. Then, the following proposition is established as demonstrated by the proof above.[13]

Proposition 2 A child allowance financed by a consumption tax can always raise fertility.

Apart from the case of an income tax, if a child allowance is financed by a consumption tax, then fertility always increases for the following reason. First, the consumption tax does not affect fertility. Therefore, under constant capital stock per capita, child allowance raises fertility. Second, consumption tax affects not only fertility but also the capital stock per capita k. An income tax reduces the household's disposable income directly and indirectly via prevention of capital accumulation because of decreased savings. This effect decreases fertility. However, this consumption tax effect is small even if an increase in population size prevents capital accumulation per capita. Therefore, if the government seeks to raise fertility always, then the child allowance is financed not by an income tax but by a consumption tax. This proposition is also obtained by Yasuoka and Goto (2011). However, Yasuoka and Goto (2011) derived this proposition in a small open economy, which capital accumulation does not consider. Consumption taxes affect not only fertility but also the capital stock per capita, which changes household income and which affects fertility indirectly. Even if this effect is included, the child allowance financed by a consumption tax can always raise fertility, as derived in this paper.

However, a child allowance should be provided to maximize not fertility but social welfare. In the next section, we examine the optimal child allowance financed by the income tax and an optimal consumption tax rate to bring about maximization of social welfare.

[12] See Appendix for a detailed proof.

[13] Even if the government levies consumption tax not only consumption but also child care z, proposition 2 is established. The budget constraint (1) changes to $(1+\tau_{ct})c_{1t} + \frac{(1+\tau_{ct+1})c_{2t+1}}{1+r_{t+1}} + (z-q)n_t = (1-\theta-\tau_t)w_t + \frac{p_{t+1}}{1+r_{t+1}}$. Then, we obtain $\frac{dn}{dq} = \frac{z(\alpha n+1+r)}{1-\alpha-\beta}\frac{\left(na_{22} - \frac{1-\beta}{1-\alpha-\beta}a_{12}\right)}{\frac{z}{1-\alpha-\beta}(\alpha n+1+r)+zn} > 0$.

4 Optimal Taxation and Child Allowance

This section explains policy effects. The model brings about an externality through a pay-as-you-go pension system, as described by van Groezen et al. (2003), van Groezen and Meijdam (2008) and Yasuoka and Goto (2011). Because of the externality, we show that the allocations in a decentralized economy differ from those of a command optimum. The social welfare function W_t is defined as[14]

$$W_t = \sum_{s=t}^{\infty} \rho^{s-t} \left(\alpha \ln c_{1s-1} + \beta \ln c_{2s} + (1-\alpha-\beta) \ln n_{s-1} \right), \tag{31}$$

where $0 < \rho < 1$ signifies the social discount factor. In a closed economy, the resource constraint per capita is shown as

$$Ak_s^{\epsilon} = c_{1s} + zn_s + \frac{c_{2s}}{n_{s-1}} + n_s k_{s+1}. \tag{32}$$

We derive the optimal allocation to maximize the social welfare subject to a resource constraint. Defining c_{1s}^*, c_{2s}^*, and n_s^* respectively as the optimal consumption of younger people, that of older people, and the optimal fertility, we derive optimal allocations subject to (32) and given $c_{1t-1}, c_{2t-1}, n_{t-1}$ and k_t as follows:[15]

$$\frac{c_{1t+1}^*}{c_{1t}^*} = \frac{\rho(1+r_{t+1})}{n_t^*}, \tag{33}$$

$$\frac{c_{2t+1}^*}{c_{1t}^*} = \frac{\beta(1+r_{t+1})}{\alpha}, \tag{34}$$

$$\frac{c_{1t}^*}{n_t^*} = \frac{\alpha(z+k_{t+1})}{1-\alpha}. \tag{35}$$

Considering allocations (3) and (4) in a decentralized economy, we derive $\frac{c_{2t+1}}{c_{1t}} = \frac{\beta(1+r_{t+1})}{\alpha}$: The saving allocation to maximize social welfare is consistent with that in a decentralized economy. In light of (32)–(35), we derive these allocations in the steady state as reduced forms as

$$c_1^* = \frac{\alpha\rho A}{\rho+\beta} k^{*\epsilon}, \tag{36}$$

$$c_2^* = \frac{\beta\rho(1+r^*)A}{\rho+\beta} k^{*\epsilon}, \tag{37}$$

$$n^* = \rho(1+r^*). \tag{38}$$

[14] As shown in van Groezen et al. (2003), if the social welfare function is defined not by the Millian criterion but by the Benthamite welfare criterion given as $W_t = \sum_{s=t}^{\infty} \rho^{s-t} \prod_{i=t}^{s} n_{s-2} u_{s-1}$, then the command optimum does not coincide with Pareto-efficient allocations. Therefore, we use the Millian criterion as van Groezen et al. (2003) use it.

[15] See Appendix for a detailed proof.

Therein, k^* denotes the capital stock per capita to maximize social welfare in the steady state and $1+r^* = A\epsilon k^{*\epsilon-1}$. Inserting these equations, Eqs. (36)–(38) into (32) in the steady state, we obtain k^* as

$$k^* = \frac{z\epsilon(\rho+\beta)}{1-\alpha-\epsilon(\rho+\beta)}. \tag{39}$$

The sign of $1-\alpha-\epsilon(\rho+\beta)$ must be positive to hold positive k^*. In the following subsection, we show an optimal child allowance financed by income tax or consumption tax and income transfer from younger people to older people.

4.1 Income Taxation

First, we derive a child allowance q^{i*} financed by income tax $\tau(\tau_c = 0)$ and the transfer for older people θ^{i*} to accommodate Eqs. (36)–(38) in the steady state. Eqs. (3) and (5) provide $c_1 = \frac{\alpha(z-q)}{1-\alpha-\beta}n$. Substituting Eqs. (36), (38), and (39) into this equation, we obtain an optimal child allowance financed by income tax q^{i*} as

$$q^{i*} = \frac{(\beta-\epsilon(\rho+\beta))z}{1-\alpha-\epsilon(\rho+\beta)}. \tag{40}$$

Considering Eqs. (3), (7), (9), (11), (36), and (39), we obtain an optimal income transfer θ^{i*} as follows

$$\theta^{i*} = \frac{(1-(1-\rho)\epsilon)(\rho+\beta)-\rho(1+\beta)}{(1-\rho)(1-\epsilon)(\rho+\beta)}. \tag{41}$$

Then, the following proposition is established.

Proposition 3 Child allowance q^{i*} and income transfer θ^{i*} should be given as (40) and (41), respectively to achieve social welfare maximization in the steady state if the government provides a child allowance financed by an income tax.

An optimal child allowance is not always positive, as shown by van Groezen and Meijdam (2008). Our paper presents examination of an optimal child allowance in closed economy, but this result is the same with Yasuoka and Goto (2011). The child allowance should be provided if the inequality $\epsilon < \frac{\beta}{\rho+\beta}$ is held. However, when this inequality is not held, the child allowance should not be provided but the government imposes a child tax. An income transfer θ^{i*} from younger people to older people should be provided, if the inequality $\epsilon < \frac{\beta}{\rho+\beta}$. Otherwise, the income transfer from older people to younger people should be adopted. This proposition was obtained already by van Groezen and Meijdam (2008), who examine a closed economy. However, our manuscript presents examination of a child allowance financed not only by an income tax but also by a consumption tax. In the following subsection, we examine the case of a consumption tax.

4.2 Consumption Taxation

We define q^{c*} as a child allowance financed by consumption tax $\tau_c(\tau = 0)$ and θ^{c*} as the transfer for older people in this case to holds Eqs. (36)–(38) in the steady state. By Eqs. (3) and (5), we obtain $c_1 = \frac{\alpha(z-q)}{(1+\tau_c)(1-\alpha-\beta)}n$. Substituting Eqs. (36) and (38) into this equation, we obtain

$$\frac{k^*}{\rho+\beta} = \frac{\epsilon(z-q^{c*})}{(1+\tau_c)(1-\alpha-\beta)}. \tag{42}$$

Substituting Eqs. (36)–(38) and $1+r^* = A\epsilon k^{*\epsilon-1}$ into (23) in the steady state, we obtain an optimal consumption tax rate as

$$\tau_c = \frac{q^{c*}\rho\epsilon(\rho+\beta)}{\alpha\rho+\beta}\frac{1}{k^*}. \tag{43}$$

An optimal child allowance financed by consumption tax q^{c*} is given by Eqs. (39), (42) and (43) as shown by

$$q^{c*} = \frac{(\beta-\epsilon(\rho+\beta))z}{\left(1+\frac{\rho(1-\alpha-\beta)}{\alpha\rho+\beta}\right)(1-\alpha-\epsilon(\rho+\beta))}. \tag{44}$$

In the steady state, inserting Eqs. (3) into (7), (22), and (36)-(38), we obtain an optimal income transfer θ^{c*}.

$$\theta^{c*} = \frac{(1-\epsilon)(\rho+\beta) - \rho\left(1+\frac{\rho(\beta-\epsilon(\rho+\beta))}{(1-\beta)\rho+\beta}\right)}{(1-\epsilon)(1-\rho)(\rho+\beta)}. \tag{45}$$

Then, the following proposition is established.

Proposition 4 Child allowance and income transfer should be given by (44) and (45), respectively, to maximize social welfare in the steady state if the government provides a child allowance financed using a consumption tax.

This proposition shows that child allowance financed by consumption tax can also bring about social welfare maximizing allocations. Similarly to the case of income taxation, based on parametric condition, the government should provide child allowance or impose a child tax. As is true also with q^{c*}, if $\epsilon < \frac{\beta}{\rho+\beta}$, a child allowance is expected to be provided unless the government imposes a tax for children. Moreover, an income transfer θ^{i*} is also provided from younger people to older people if $\epsilon < \frac{\beta}{\rho+\beta}$. Then, the proof shown above establishes the following proposition.

Proposition 5 Whichever the kind of taxation, if $\epsilon < \frac{\beta}{\rho+\beta}$, then the government should provide a child allowance and income transfer from younger people to older people to maximize social welfare in the steady state.

The child allowance and income transfer directly affect not only fertility and consumption but also the capital stock per capita. The change of capital stock alters fertility and consumption indirectly. With $\epsilon < \frac{\beta}{\rho+\beta}$, a direct effect dominates the indirect effect; then the direct effects of child allowance and income transfer can bring about socially optimal allocations. Alternatively, an indirect effect created by a child tax and income transfer from older people to younger people changes the capital stock. It can bring about socially optimal allocations.

Finally, we compare (40) with (44), which reveals that the absolute value of (40) is less than that of (44): $|q^{i*}| > |q^{c*}|$. Then, we obtain the following proposition.

Proposition 6 Child allowance financed by consumption tax is smaller than that by income tax if the government is expected to provide a child allowance to maximize social welfare in the steady state.

Why is this proposition established? Optimal fertility is given as shown in (38). The child allowance increases fertility and decreases capital stock per capita because of an increase in the population size. However, the consumption tax reduces capital stock to less than the case of an income tax. An income tax reduces household's saving and then capital stock decreases. Income tax reduces the fertility and the fertility diverges largely from optimal fertility. Therefore, so a large amount of child allowance must be provided if the child allowance is financed by income tax. This proposition is not obtained by van Groezen et al. (2003), van Groezen and Meijdam (2008) and Yasuoka and Goto (2011). This proposition is derived originally herein.

5 Conclusions and Remarks

This paper presented an examination, using an endogenous fertility model, of how a child allowance for younger people is to be collected. Concretely, this paper presented a description of how the child allowance affects fertility. In addition, this paper examined the optimal tax policy to maximize social welfare. The analyses described herein yielded the following results.

First, the child allowance financed by a child allowance can not always raise fertility as shown by earlier studies. However, a child allowance financed by a consumption tax can always raise fertility in spite of a closed economy. Therefore, we can find that the effect of child allowance for fertility changes based on how it is financed.

Second, this paper has presented derivation of the allocations to maximize social welfare presented an examination of the optimal taxation to achieve optimal allocations. The optimal child allowance and an optimal income transfer from younger people to older people differs between the case of an income tax and the case of a consumption tax. An optimal policy to achieve social welfare maximization differs according to the mode of taxation.

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Appendix

The sign of $a_{11}a_{22} - a_{12}a_{21}$

$$\begin{aligned} a_{11}a_{22} - a_{12}a_{21} &= \frac{(1-\alpha-\beta)(1-\theta)(1-\epsilon)Ak^{\epsilon}}{n}\left(1 - \frac{\epsilon(1-\theta)(1-\epsilon)Ak^{\epsilon-1}}{n}\right) \\ &+ \frac{(1-\alpha-\beta)(1-\theta)(1-\epsilon)^2 Ak^{\epsilon}}{n^2}\left(\frac{\theta}{\epsilon}n + (1-\theta)\epsilon Ak^{\epsilon-1}\right) \\ &= \frac{(1-\alpha-\beta)(1-\theta)(1-\epsilon)Ak^{\epsilon}}{n}\frac{\epsilon+(1-\epsilon)\theta}{\epsilon} > 0. \end{aligned}$$

The sign of (19)

$$\begin{aligned} (\alpha+\beta)na_{22}\frac{\alpha}{1-\alpha-\beta}a_{12} &= \frac{(\alpha+\beta)\epsilon + \alpha(1-\epsilon)\theta}{\epsilon}n - \beta(1-\epsilon)(1-\theta)\epsilon Ak^{\epsilon-1} \\ &= n\left(\frac{(\alpha+\beta)\epsilon + \alpha(1-\epsilon)\theta}{\epsilon} - \frac{\beta\epsilon}{k}\frac{z - \frac{\theta(1-\alpha-\beta)(1-\epsilon)}{\epsilon}k}{1-\alpha-\beta}\right) \\ &= \frac{(\alpha+\beta)\epsilon + \alpha(1-\epsilon)\theta}{\epsilon} - \beta\epsilon - \frac{\beta\epsilon(1-\beta)z}{1-\alpha-\beta}\frac{1}{k} \\ &= \frac{1}{\epsilon}\left((\alpha+\beta)\epsilon + \alpha(1-\epsilon)\theta - \beta\epsilon^2 - (1-\beta)\epsilon(\epsilon+\theta(1-\epsilon))\right). \end{aligned}$$

The bracket of this equation is positive if Eq. (21) holds.

The sign of (30)

$$\begin{aligned} na_{22} - \frac{1-\beta}{1-\alpha-\beta}a_{12} &= n + \frac{(1-\beta)(1-\epsilon)\theta}{\epsilon}n - \beta(1-\epsilon)(1-\theta)\epsilon Ak^{\epsilon-1} \\ &= n\left(\frac{\epsilon+(1-\beta)(1-\epsilon)\theta}{\epsilon} - \frac{\beta\epsilon}{k}\frac{z - \frac{(1-\alpha-\beta)(1-\epsilon)\theta}{\epsilon}k}{1-\alpha-\beta}\right) \\ &= n\left(\frac{\epsilon+(1-\beta)(1-\epsilon)\theta}{\epsilon} - \frac{\beta\epsilon}{k}\left(k + \frac{(1-\beta)z}{1-\alpha-\beta}\right)\right) \\ &= n\left((1-\epsilon)(1-(1-\beta)\theta) + \frac{(1-\beta)(1-\epsilon)\theta}{\epsilon}\right) > 0. \end{aligned}$$

Command Optimum Allocations

We set the Lagrange equation as follows

$$\begin{aligned} L = &\sum_{s=t}^{\infty}\rho^{s-t}\left(\alpha\ln c_{1s-1} + \beta\ln c_{2s} + (1-\alpha-\beta)\ln n_{s-1}\right) \\ &+ \sum_{s=t}^{\infty}\lambda_{s-1}\left(c_{1s-1} + zn_{s-1} + \frac{c_{2s-1}}{n_{s-2}} + n_{s-1}k_s - Ak_{s-1}^{\epsilon}\right), \end{aligned}$$

where λ_{i-1} denotes a Lagrange multiplier. We obtain the first order conditions as

$$\frac{\partial L}{\partial c_{1t}} = \frac{\alpha\rho}{c_{1t}} + \lambda_t = 0, \tag{46}$$

$$\frac{\partial L}{\partial c_{2t+1}} = \frac{\beta\rho}{c_{2t+1}} + \frac{\lambda_{t+1}}{n_t} = 0, \tag{47}$$

$$\frac{\partial L}{\partial n_t} = \frac{(1-\alpha-\beta)\rho}{n_t} + \lambda_t(z+k) - \frac{\lambda_{t+1}c_{2t+1}}{n_t^2} = 0, \tag{48}$$

$$\frac{\partial L}{\partial k_t} = \lambda_{t-1}n_{t-1} - \lambda_t \epsilon A k_t^{\epsilon-1} = 0, \tag{49}$$

$$\frac{\partial L}{\partial \lambda_t} = c_{1t} + zn_t + \frac{c_{2t}}{n_{t-1}} + n_t k_{t+1} - Ak_t^{\epsilon} = 0. \tag{50}$$

Substituting Eqs. (46) and (47) into (49), we obtain Eq. (34). Substituting Eqs. (46) and (47) into (48), we obtain Eq. (35). Considering

$$\frac{\partial L}{\partial c_{1t+1}} = \frac{\alpha\rho^2}{c_{1t+1}} + \lambda_{t+1} = 0, \tag{51}$$

Eqs. (46) and (51), we obtain (33). In the steady state, Eq. (33) changes to (38). Inserting $c_2^* = \frac{\beta(1+r)}{\alpha}c_1^*$ and $n^* = \rho(1+r)$ into (32) in the steady state, we obtain c_1^*, as shown by (36). Then, Eq. (36) into $c_2^* = \frac{\beta(1+r)}{\alpha}c_1^*$, we obtain c_2^*.

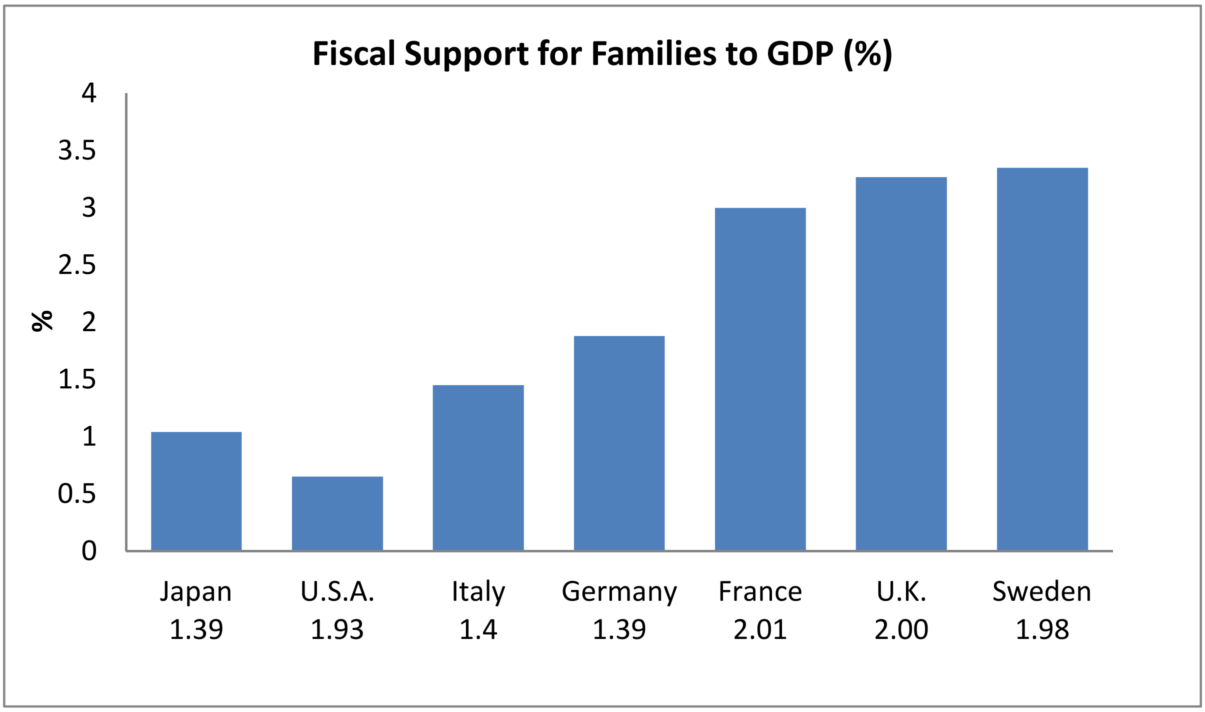

Fig. 1: Fertility (below the country) and Fiscal Support for Family (share of Gross Domestic Product) (Data: OECD Social Expenditure Database (November 2008), A 2012 Declining Birthrate White Paper (2012), Demographic Yearbook (UN) and Vital Statistics in Japan (Ministry of Health, Labour and Welfare (in Japan).) Data of Fiscal Support for Families are those of 2007. Fiscal Support for Family includes benefits in kind (day-care/home help and other benefits in kind) and cash benefits (family allowance, maternity and parental leave and other cash benefit). Data of the total Fertility Rate are those of 2010.)

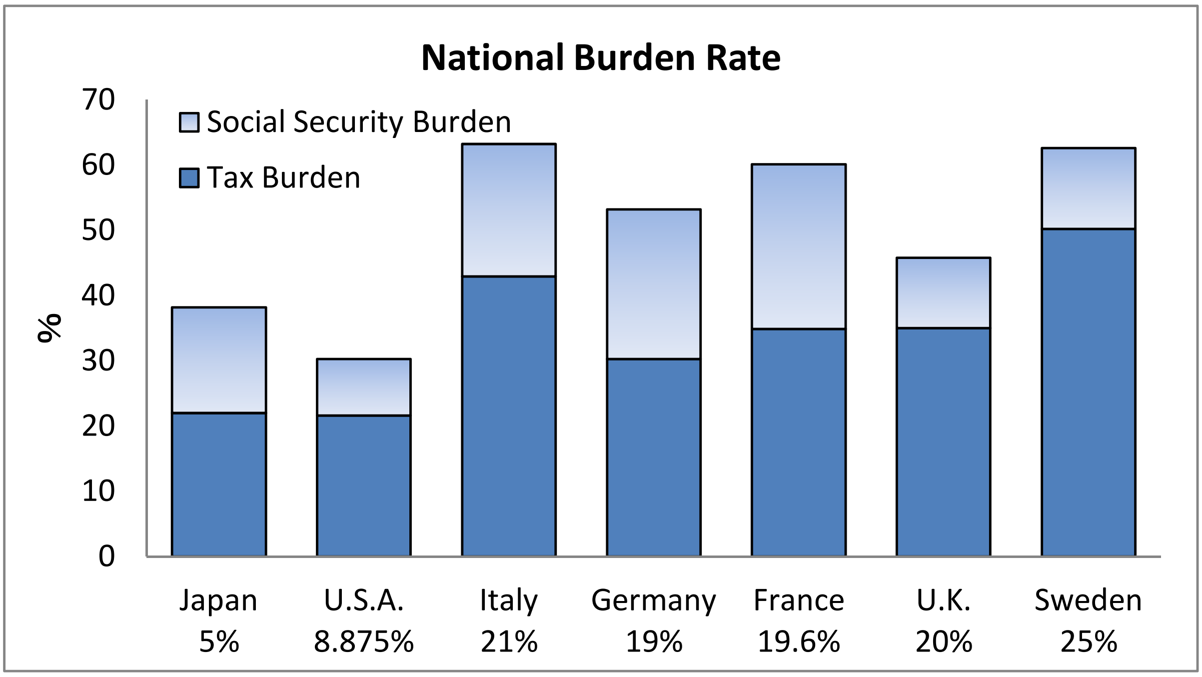

Fig. 2: National Burden Rate (Data: National Accounts (Cabinet Office, Government of Japan), Consumption Tax Rate in foreign countries (Ministry of Finance Japan), National Accounts (OECD), Revenue Statistics (OECD). Data are for 2009. The percentage below country name denotes the consumption tax rate. The consumption tax rate for the U.S.A. is that of New York City.)

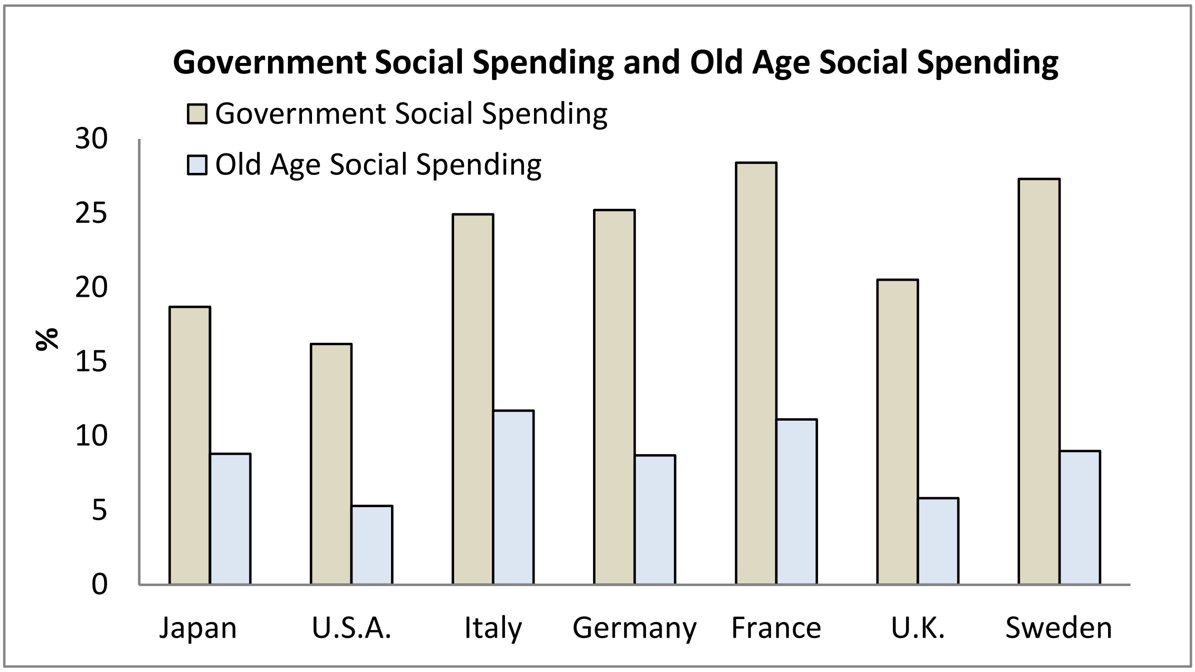

Fig. 3: Government Social Spending and Old Age Social Spending (data show public social expenditure as a percentage of GDP. (Data: Social Expenditure: Aggregated data, OECD Social Expenditure Statistics (database)))