Public Investment, Life Expectancy and Income Growth

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Abstract

Based on individual occupational choice in the model including a production function with public investment, this paper presents an examination of how public investment affects the dynamics. Individuals work as skilled laborers or unskilled laborers. As in the model described by Caselli (1999), educational costs are necessary to work as a skilled laborer. Results show that life expectancy determines whether income growth occurs or not. Public investment can bring about income growth if life expectancy is sufficiently high. However, with low probability, the government can not bring about income growth with an increase in public investment.

JEL Classification: H54, E20

Key Words: Public investment, Life expectancy, Occupational choice, Economic growth

1 Introduction

Based on simple OLG, we discuss occupational choice, either skilled or unskilled, in addition to public investment with life expectancy. Especially, we address not only public investment but also life expectancy against dynamics. Many papers describe studies of capital accumulation by government. Barro (1990), Barro and Sala-i-Martin (1992), Futagami and Morita and Shibata (1993) examine public capital accumulation and growth. Glomm and Ravikumar (1997) show public accumulation and growth with human capital investment. Turnovsky (1997) also discusses public capital accumulation and growth and the difference between a socially planned economy and a decentralized economy. Yakita (2008) discusses an endogenized fertility rate and an aging economy including public accumulation. Public investment in these earlier studies serves a role in increasing productivity.

Apart from public capital accumulation, many papers have described studies of educational choice. Maoz and Moav (2000) examine skill acquisition and inequality including intergenerational mobility. Caselli (1999) and Galor and Moav (2000) introduce an idea that new technology prohibits people from learning new skills. Chen (2010) uses an overlapping generations model with life expectancy and educational choice.

Based on Chen (2010), this paper presents the derivation of some results. Depending on public investment and life expectancy, it is decided whether income growth continues or the income level converges to a constant level. The economy can not escape from a poverty trap if life expectancy is low. Chen (2010) shows that increased life expectancy reduces fertility. Subsequently, capital per capita increases. Therefore the economy can avoid a poverty trap. Different from Chen (2010), this paper presents consideration of public investment, which raises productivity. However, with low life expectancy, public investment can not bring about income growth. Therefore, both public investment and life expectancy should be regarded as continued income growth.

The remainder of this paper is organized as follows. Section 2 introduces the model. Section 3 presents a description of equilibrium and macroeconomic dynamics and derives the conditions under which income growth continues. Lastly, we summarize our manuscript.

2 The Model

The model economy is based on a two-period (young and old) overlapping generations model. This economy has agents of three types: households, firms, and a government.

2.1 Households

Households experience two periods: young and old. During the young period, each household supplies labor to earn labor income. This economy accommodates labor of two types: skilled labor and unskilled

labor. Education costs must be incurred to become a skilled laborer, as assumed by Caselli (1999), Meckl and Zink (2004), Miyake, Muro, Nakamura and Yasuoka (2009), and Chen (2010). That cost is assumed as σ. Herein, w_t^s denotes the wage rate of skilled labor. The government imposes labor income taxation on the wage income of skilled labor to provide public investment. Each household allocates its labor income between consumption in the young period and saving. Consequently, we obtain the following budget constraint:

$$c_{1t}^s + \frac{c_{2t+1}^s}{R_{t+1}} = (1 - \tau)w_t^s - \sigma, \tag{1}$$

$$c_{1t}^u + \frac{c_{2t+1}^u}{R_{t+1}} = w_t^u. \tag{2}$$

Indexes s and u respectively denote skilled labor and unskilled labor. Also, c_{1t} and c_{2t+1} respectively denote consumption in the young period and old period. w_t^u denotes the wage rate of unskilled sector. R_{t+1} signifies an interest rate for annuitized savings. τ signifies labor income tax rate ($0 < \tau < 1$). Finally, t denotes the period. A household's utility function u_t is given as shown below.

$$u_t = \alpha \ln c_{1t} + p(1 - \alpha) \ln c_{2t+1}, \ 0 < \alpha < 1, \ 0 < p < 1 \tag{3}$$

Therein, p denotes the probability that the individual lives during the old period. These savings are allocated among older living people if the individual dies. This is annuitized wealth. The optimal allocations at skilled labor are determined as

$$c_{1t} = \frac{\alpha}{\alpha + p(1 - \alpha)} \left((1 - \tau)w_t^s - \sigma \right), \tag{4}$$

$$c_{2t+1} = \frac{p(1 - \alpha)R_{t+1}}{\alpha + p(1 - \alpha)} \left((1 - \tau)w_t^s - \sigma \right). \tag{5}$$

If a worker is an unskilled laborer, then

$$c_{1t} = \frac{\alpha}{\alpha + p(1 - \alpha)} w_t^u, \tag{6}$$

$$c_{2t+1} = \frac{p(1 - \alpha)R_{t+1}}{\alpha + p(1 - \alpha)} w_t^u. \tag{7}$$

2.2 Firm

This paper assumes the production function shown below[1].

$$Y_t = AK_t^\theta (G_t L_t)^{1-\theta} + B(1 - L_t), \ 0 < \theta < 1, \ 0 < A, \ 0 < B \tag{8}$$

Therein, Y_t denotes the aggregate output. G_t and K_t respectively denote public investment and capital stock. L_t denotes the skilled labor amount. Considering that the population size of each generation

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[1] Some papers consider a production function with public investment. For example, Barro (1990) assumed $Y = K^\theta (GL)^{1-\theta}$. In addition, Caselli (1999) assumed that not only labor but also capital stock is inputted as a productive factor in unskilled sector. Neither Caselli (1999) nor Chen (2010) considered public investment.

is unity, the unskilled labor amount is shown by $1 - L_t$. With a perfectly competitive market, profit maximization reduces the following equations, as

$$w_t^s = A(1-\theta)G_t^{1-\theta}\left(\frac{K_t}{L_t}\right)^\theta, \tag{9}$$

$$w_t^u = B. \tag{10}$$

The interest rate is shown as

$$1 + r_t = A\theta\left(\frac{K_t}{G_t L_t}\right)^{\theta-1}. \tag{11}$$

It is noteworthy that

$$R_{t+1} = \frac{1 + r_{t+1}}{p}. \tag{12}$$

Capital stock is assumed to be fully depreciated in one period.

2.3   Government

The government imposes labor income taxation at a tax rate τ on skilled labor to provide public investment. Then, the government budget constraint is presented as

$$G_t = \tau w_t^s L_t. \tag{13}$$

3   Equilibrium

This section presents derivation of the equilibrium of this model economy. If workers move between two sectors freely, then the indifference condition is described as

$$w_t^s = \frac{B + \sigma}{1 - \tau} \equiv \hat{w}. \tag{14}$$

Considering (9), (13), and (14), L_t is given as

$$L_t = \left(\frac{(1-\tau)\tau^{\frac{1-\theta}{\theta}}(1-\theta)^{\frac{1}{\theta}}A^{\frac{1}{\theta}}K_t}{B+\sigma}\right)^{\frac{\theta}{2\theta-1}}. \tag{15}$$

Intuitively, an increase in K_t raises L_t because an increase in w_t^s brings about an increase in the amount of skilled labor. Therefore, we assume $\theta > \frac{1}{2}$. Additionally, we assume the total population size in each generation as unity. Then, the dynamics of capital stock at $L_t < 1$ is derived as shown below:

$$K_{t+1} = \frac{p(1-\alpha)}{\alpha + p(1-\alpha)}\left(((1-\tau)w_t^s - \sigma)L_t + B(1-L_t)\right) = \frac{p(1-\alpha)B}{\alpha + p(1-\alpha)}. \tag{16}$$

In contrast, an increase in K_t raises L_t. Therefore, the dynamics of capital stock at $L_t^s = 1$ is derived as

$$K_{t+1} = \frac{p(1-\alpha)}{\alpha + p(1-\alpha)}\left((1-\tau)\tau^{\frac{1-\theta}{\theta}}(1-\theta)^{\frac{1}{\theta}}A^{\frac{1}{\theta}}K_t - \sigma\right). \tag{17}$$

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Then, w_t^s and $1 + r_t$ at $L_t = 1$ are

$$w_t^s = \tau^{\frac{1-\theta}{\theta}} (1-\theta)^{\frac{1}{\theta}} A^{\frac{1}{\theta}} K_t, \tag{18}$$

$$1 + r_t = \tau^{\frac{1-\theta}{\theta}} \theta (1-\theta)^{\frac{1-\theta}{\theta}} A^{\frac{1}{\theta}}. \tag{19}$$

Therefore, calculating $Y_t = (1 + r_t)K_t + w_t^s L_t$, we obtain $Y_t = \tau^{\frac{1-\theta}{\theta}}(1-\theta)^{\frac{1-\theta}{\theta}} A^{\frac{1}{\theta}} K_t$. The capital stock $\hat{K}$ which brings about $w_t^s = \hat{w}$ and $L_t = 1$ is

$$\hat{K} = \frac{B + \sigma}{(1-\tau)\tau^{\frac{1-\theta}{\theta}}(1-\theta)^{\frac{1}{\theta}} A^{\frac{1}{\theta}}}. \tag{20}$$

Then, we can depict the dynamics of K_t as shown below.

[Insert Fig. 1 around here.]

There exist dynamics of three types. The first is that income growth occurs for any K_0 (Fig. 1-1). The second is that income growth occurs or does not occur for given K_0 (Fig. 1-2). If K_0 is greater than $\tilde{K}$, income growth occurs. The last is that no income growth occurs (Fig. 1-3). Life expectancy p determines which case is adopted.

The condition that the slope of K_{t+1} shown by (17) is greater than unity is shown as

$$p > \frac{\alpha}{(1-\alpha)\left((1-\tau)\tau^{\frac{1-\theta}{\theta}}(1-\theta)^{\frac{1}{\theta}} A^{\frac{1}{\theta}} - 1\right)} \equiv p_1, \tag{21}$$

where $(1-\tau)\tau^{\frac{1-\theta}{\theta}}(1-\theta)^{\frac{1}{\theta}} A^{\frac{1}{\theta}} > 1$ is assumed. Otherwise, no p exists that holds this inequality. The condition that $\hat{K}$ has no point of intersection with the 45-degree line ($K_{t+1} = K_t$) is shown as

$$p > \frac{\alpha\left(\frac{\sigma}{B} + 1\right)}{(1-\alpha)\left((1-\tau)\tau^{\frac{1-\theta}{\theta}}(1-\theta)^{\frac{1}{\theta}} A^{\frac{1}{\theta}} - \frac{\sigma}{B} - 1\right)} \equiv p_2 \tag{22}$$

where $(1-\tau)\tau^{\frac{1-\theta}{\theta}}(1-\theta)^{\frac{1}{\theta}} A^{\frac{1}{\theta}} > 1 + \frac{\sigma}{B}$ is assumed. Otherwise, no p exists that is true for this inequality. We obtain $0 < p_1 < p_2$. Therefore, with $p_2 < p$, the dynamics of K_{t+1} is depicted as Fig. 1-1. With $p_1 < p < p_2$, the dynamics of K_{t+1} is depicted as Fig. 1-2. However, with $p < p_1$, the dynamics of K_{t+1} is depicted as Fig. 1-3. Then, the following proposition is established.

Proposition 1 With $p_2 < p$, income growth continues for any initial capital stock. With $p_1 < p < p_2$, income growth continues for initial capital stock that is greater than $\tilde{K}$. With $p < p_1$, no income growth exists and the economy remains in a poverty trap.

This proposition shows that life expectancy, i.e. aging, determines whether income growth occurs or not. An increase in p, which signifies an aging society, raises the saving and capital stock. An increase in capital stock raises the wage rate in skilled labor, which increases capital stock. This paper presents the steady state with K^* as a poverty trap because no increase in income occurs.

Next, we examine whether an increase in τ can decrease $\hat{K}$. If the economy stays in the case given in Fig. 1-3, then a decrease in $\hat{K}$ brings about the economy given by Fig. 1-1, which continues increasing income. Defining $F(\tau) = (1-\tau)\tau^{\frac{1-\theta}{\theta}}$, $F(\tau)$ is maximized at $\tau = 1 - \theta$. Therefore, this substitutes into p_2, and we obtain

$$\frac{\alpha\left(\frac{\sigma}{B} + 1\right)}{(1-\alpha)\left(\theta(1-\theta)^{\frac{2-\theta}{\theta}}A^{\frac{1}{\theta}} - \frac{\sigma}{B} - 1\right)} \equiv \tilde{p}_2. \tag{23}$$

Then, the following proposition is established.

Proposition 2 With $p < \tilde{p}_2$, an increase in τ can not bring about income growth.

An increase in τ increases public investment, which raises productivity. However, this increase reduces capital accumulation, which thereby decreases productivity. Capital accumulation is not large if life expectancy p is low. Therefore, a decrease in capital accumulation greatly reduces productivity.

In developed countries, we regard p as small. Even if the government increases public investment to pull up the income level, an income level converges to a poverty trap as long as the government levies an income tax. In fact, p must become large to bring about income growth, such as a government providing medical service appropriately.

4   Concluding and Remarks

This paper described a model with public investment and illustrated how an increase in public investment affects capital stock, the amount of skilled labor, and the wage rate. First, when the government provides public investment, the government can achieve income growth if life expectancy is high. However, with low life expectancy, public investment does not engender income growth, meaning that the income level converges to a constant level: a poverty trap. Therefore, the government should provide not only public investment but also medical services to increase life expectancy and escape from the poverty trap.

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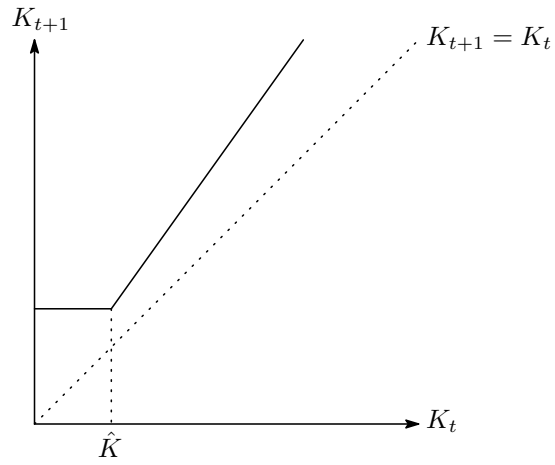
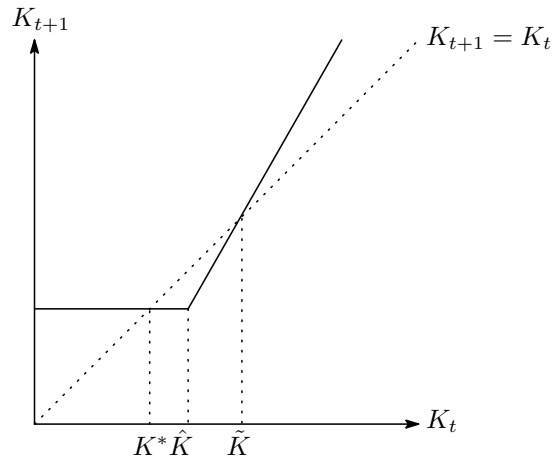Fig. 1-1: Dynamics of K_t (Income Growth).

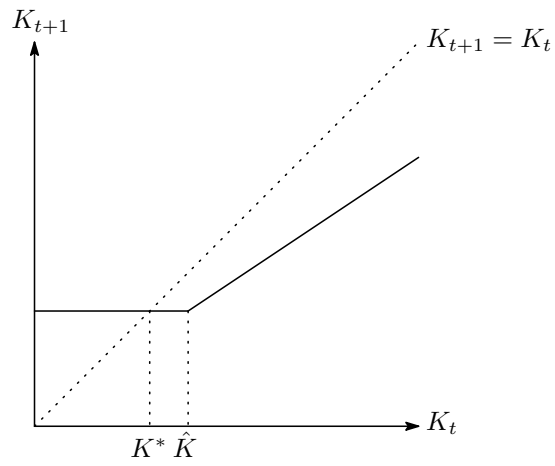
Fig. 1-2: Dynamics of K_t (Income Growth or No Income Growth).

Fig. 1-3: Dynamics of K_t (No Income Growth).

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